Spring 2015

MAC2313 Test 2 A

(5 pts) 1. If $g(x, y, z) = 2xy + yz^2$, then $\nabla g(2, 1, -1)$ is parallel to which of the following vectors?

$\nabla g = \langle 2y, 2x+z^2, 2yz \rangle$ $\quad \nabla g(2,1,-1) = \langle 2, 5, -2 \rangle \times -2 = \langle -4, -10, +4 \rangle$

A. $\langle 1, 2, 6\rangle$ B. $\langle 0, 3, 1\rangle$ $\boxed{\text{C. } \langle -4, -10, 4\rangle}$ D. $\langle 0, 2, 4\rangle$ E. $\langle 3, -5, 1\rangle$

(5 pts) 2. The tangent plane to the graph of $z = x^2 - y^2$ at the point $(x, y) = (2, 1)$ is given by:

$f_x = 2x \quad f_y = -2y \quad f_x(2,1) = 4 \quad f_y(2,1) = -2 \quad f(2,1) = 3$

A. $L(x, y) = 4x - 2y + 3$ B. $L(x, y) = 2x - 4y + 3$ C. $L(x, y) = 2x - 2y - 3$

D. $L(x, y) = 3x - 2y + 4$ $\boxed{\text{E. } L(x, y) = 4x - 2y - 3}$

$L(x,y) = 4(x-2) - 2(y-1) + 3$
$= 4x - 2y - 3$

(5 pts) 3. If $f(x, y) = \ln(4x^2 - y)$, how many of the following are true?

i. $D = \{\ (x, y) \mid y \geq 4x^2\ \}$ $\quad 4x^2 - y > 0 \Rightarrow y < 4x^2$

F

ii. $R = (-\infty, \infty)$ T Take $x=0$, $y<0$

iii. The domain is contained in the x, y- plane. T

iv. The point $(-2, 16)$ is in the domain. $\quad 4(-2)^2 - 16 = 4 \cdot 4 - 16 = 16 - 16 = 0$ X

F

A. 0 B. 1 $\boxed{\text{C. } 2}$ D. 3 E. 4

(5 pts) 4. The directional derivative of $f(x,y) = x^2y$ at the point $(5,1)$ in the direction of $\langle 3,4 \rangle$ $\|v\| = 5$
is given by: $\nabla f = \langle 2xy, x^2 \rangle$ $\nabla f(5,1) = \langle 10, 25 \rangle$ $u = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$D_u f(P) = \langle 10, 25 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = 6 + 20 = 26$

A. -23 B. 0 C. 5 $\boxed{\text{D. } 26}$ E. none of the above

(5 pts) 5. Let $f(x,y)$ be a function such that $f_x = 3y^2$ and $f_y = 6xy$; in addition let $x = 2s + 3t$ and $y = s - t$. The derivative of $f(x,y)$ with respect to s at $(s,t) = (1,0)$ is given by:

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s} = 3y^2 \cdot 2 + 6xy \cdot 1 = 6y^2 + 6xy$ $(s,t) = (1,0) \Rightarrow x = 2, y = 1$

A. -2 B. 0 C. 6 D. 12 $\boxed{\text{E. } 18}$

$\frac{\partial f}{\partial s}\Big|_{(x,y)=(2,1)} = 6 + 12 = 18$

(5 pts) 6. If $f(x,y)$ is differentiable at (a,b) then

$$f(x,y) \approx f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b).$$

$\boxed{\text{A. True}}$ B. False

(5 pts) 7. What is the value of the following limit:

$$\lim_{(x,y)\to(2,3)} \frac{\sqrt{3x} - \sqrt{2y}}{3x - 2y} ?$$

A. $-\sqrt{6}/6$ B. 0 C. $\sqrt{6}/6$ $\boxed{\text{D. } \sqrt{6}/12}$ E. does not exist

$$\lim_{(x,y)\to(2,3)} \frac{\cancel{\sqrt{3x} - \sqrt{2y}}}{\cancel{(\sqrt{3x} - \sqrt{2y})}(\sqrt{3x} + \sqrt{2y})} = \lim_{(x,y)\to(2,3)} \frac{1}{\sqrt{3x} + \sqrt{2y}} = \frac{1}{\sqrt{6} + \sqrt{6}} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

(5 pts) 8. If $f(x,y) = (2x^2+3y^2)^2$, then $\frac{\partial^2 f}{\partial x^2}(0,1)$ is equal to:

A. 12 $\boxed{\text{B. 24}}$ C. 36 D. 48 E. 64

$\frac{\partial f}{\partial x} = 2(2x^2+3y^2)(4x) = 8x(2x^2+3y^2) = 16x^3 + 24xy^2 \Rightarrow \frac{\partial^2 f}{\partial x^2} = 48x^2 + 24y^2\Big|_{(0,1)} = 24$

(5 pts) 9. <u>How many</u> of the following are true?

i. If $f(x,y)$ is continuous at (a,b) then the limit of the function as $(x,y) \to (a,b)$ exists. T

ii. If the limit as $(x,y) \to (a,b)$ of $f(x,y)$ exists, then the function is continuous at (a,b). F

Also need $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

iii. If (a,b) is a boundary point for the domain of $f(x,y)$, then the limit as $(x,y) \to (a,b)$ of $f(x,y)$ is equal to $f(a,b)$. F

Depending on domain, f may not be defined at (a,b); even if it is, f may not be continuous.

iv. If (a,b) is a boundary point for the domain of $f(x,y)$, then the function is defined at that point. F Depending on domain, f may not be defined at boundary points.

A. 0 $\boxed{\text{B. 1}}$ C. 2 D. 3 E. 4

(5 pts) 10. If $f(x,y)$ is differentiable at (a,b) then it is continuous at (a,b).

$\boxed{\text{A. True}}$ B. False

(5 pts) 11. The tangent plane to the ellipsoid $x^2 + y^2/4 + z^2/9 = 25$ at the point $(3, 8, 0)$ has a normal vector given by: $\nabla f = \langle 2x, \frac{y}{2}, \frac{2z}{9} \rangle$ $\nabla f(3,8,0) = \langle 6, 4, 0 \rangle$

A. $\langle 1, 2, 6 \rangle$ B. $\langle 1, 2, 3 \rangle$ $\boxed{\text{C. } \langle 6, 4, 0 \rangle}$ D. $\langle 4, 6, 0 \rangle$ E. $\langle 2, 1, 0 \rangle$

(5 pts) 12. For a given function $f(x, y)$ the following are true: $f_x(a, b) = f_y(a, b) = 0$, $f_{xx}(a, b) = 3$, $f_{yy}(a, b) = -2$, and $f_{xy}(a, b) = 3$. Which of the following are true?

$D = f_{xx} f_{yy} - f_{xy}^2 = 3 \cdot (-2) - 3^2 = -6 - 9 = -15 < 0$

A. The function has a local max at (a, b).

B. The function has a local min at (a, b).

$\boxed{\text{C.}}$ The function has a saddle point at (a, b).

D. The second derivative test fails at (a, b).

E. None of the above are true.

(5 pts) 13. Assume that f is defined on an open set D of R^2 and f_{xy} and f_{yx} are continuous throughout D. Then $f_{xy} = f_{yx}$ at all points in D.

The above Theorem is known as:

A. The fundamental theorem of partial derivatives

B. Stoke's Theorem

C. The fundamental theorem of functions of two variables

$\boxed{\text{D.}}$ Clairaut's Theorem

E. none of the above

Bonus (5 pts) 14. If $f(x, y) = 2x + 3y$ which of the following vectors point in the direction in which the function is increasing most rapidly at the point $(0, 0)$?

$\nabla f = \langle 2,3 \rangle \quad \nabla f(0,0) = \langle 2,3 \rangle$

$\boxed{\text{A. } \langle 2, 3\rangle}$ B. $\langle -2, -3\rangle$ C. $\langle 2, -3\rangle$ D. $\langle 2, -3\rangle$ E. $\langle 3, -2\rangle$

Bonus (5 pts) 15. The function $f(x, y) = x^2 + 2xy - y^2 + 3x - 3y$ has how many critical points?

A. 0 $\boxed{\text{B. } 1}$ C. 2 D. 3 E. none of the above

$f_x = 2x + 2y + 3 = 0 \Rightarrow$ (add together)

$f_y = 2x - 2y - 3 = 0$

$4x = 0 \Rightarrow x = 0$

Then $2y + 3 = 0 \Rightarrow 2y = -3 \Rightarrow y = -3/2$

Critical pt $(0, -3/2)$

Name: ______________________ UF-ID: ________________ Section: ______________

(7 pts) 1. For $f(x,y) = 2x^2y - 3y^2$ at the point $(2,-1)$, give a vector which points in the direction in which the function is not changing.

(5 pts) 2. Consider the function $f(x,y)$ where x and y are functions of the single variable t. Give the chain rule for calculating the derivative of f with respect to t.

(5 pts) 3. Consider the function $g(x,y,z)$ where x, y, and z are functions of the variables s and t. Give the chain rule for calculating the derivative of g with respect to s.

(8 pts) 4. The volume of a right circular cone with radius r and height h is given by $V = (1/3)\pi r^2 h$. Approximate the change in the volume of the cone as the radius changes from $r = 1.0$ to $r = 1.1$ and the height changes from $h = 3.0$ to $h = 2.9$.

(10 pts) 5. Find and classify the critical points for the function $f(x, y) = 3x - x^3 - 3xy^2$. (Hint: there are four critical points.)