UF UNIVERSITY of FLORIDA
Department of Mathematics

MAC 2313
Exam 2A
Summer C 2015

A. Sign your bubble sheet on the back at the bottom <u>in ink</u>.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 2, 1.

1 ● 3 4 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) This test consists of 15 multiple choice questions, worth 5 points each, plus two sheets (four pages) of free response questions worth 35 points. The test is counted out of 110 points, and there are 10 bonus points available.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture <u>I.D. with a legible signature</u>.

3) The answers will be posted in Sakai (e-Learning) within one day after the exam. Your instructor will return your tearoff sheet with your exam score in class. Your score will also be posted in e-Learning within one week of the exam.

NOTE: Be sure to bubble the answers to questions 1−15 on your scantron.

Questions 1 – 15 are worth 5 points each.

1. Calculate the limit:

$$\lim_{(x,y)\to(2,1)} \frac{xy-y^2}{e^{y^2+2x+1}}$$

a. e b. e^{-2} c. e^{-6} d. e^{-8} e. The limit does not exist

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2. Let (a,b) be a point in the domain of the real-valued function $f(x,y)$ and let $L \in \mathbb{R}$. Which of the following are true?

P. If $f(x,y)$ is continuous at (a,b), then $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

Q. If there are two different paths along which $\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$, then $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exists and equals L.

R. If there are infinitely many paths along which $\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$, then $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exists and equals L.

S. If f is continuous at (a,b) and both partial derivatives $f_x(a,b)$ and $f_y(a,b)$ exist, then f is differentiable at (a,b).

a. P only b. P, Q, and R c. P and R

d. P, R, and S e. R and S

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3. If $g(x, y, z) = xy^2 + 5yz$, then $\nabla g(4, -2, 1)$ is parallel to which of the following vectors?

a. $\langle 8, 22, 20\rangle$ b. $\langle -8, 22, 20\rangle$ c. $\langle 8, 2, -1\rangle$

d. $\langle 8, 2, 1\rangle$ e. $\langle 5, 0, -2\rangle$

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4. If $f(x, y)$ is a differentiable function, and $\mathbf{r}(t)$ is a parametrization of the level curve at $f(a, b)$ such that $\mathbf{r}(t_0)$ represents the point $(a, b, f(a, b))$, which of the following statements are true?

P. If $\nabla f(a, b) \neq \mathbf{0}$, then $\nabla f(a, b)$ is orthogonal to $\mathbf{r}'(t_0)$.

Q. If $\mathbf{u} = \langle u_1, u_2\rangle$ is any vector, the directional derivative in the direction of $\mathbf{u}$ is given by $D_{\mathbf{u}} f(a, b) = \nabla f(a, b) \cdot \langle u_1, u_2\rangle$.

R. The equation of the tangent plane to f at (a, b) is given by

$$L(x, y) = f(a, b) + \nabla f(a, b) \cdot \langle x - a, y - b\rangle$$

S. The directional derivative of f at (a, b) in the direction given by $\mathbf{u} = \langle f_y(a, b), -f_x(a, b)\rangle$ is zero.

a. P only b. P and S only c. P, Q, and S only

d. P, R, and S only e. Q and S only

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5. What is the domain of the function $f(x, y) = \ln(|x - y|)$?

a. $\{(x, y)|x > y\}$ b. $\mathbf{R^2}$ c. $\{(x, y)|x \neq y\}$

d. $\{(x, y)|x - y < 0\}$ e. None of the above

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6. Given the function $f(x,y) = \tan\left(\dfrac{x}{y}\right)$, find f_x and f_y.

a. $f_x = \dfrac{\sec^2\left(\frac{x}{y}\right)}{y}$ and $f_y = \dfrac{-x\cos^2\left(\frac{x}{y}\right)}{y^2}$

b. $f_x = \dfrac{-\sec^2\left(\frac{x}{y}\right)}{y}$ and $f_y = \dfrac{-x}{y^2\cos^2\left(\frac{x}{y}\right)}$

c. $f_x = \dfrac{y}{\cos^2\left(\frac{x}{y}\right)}$ and $f_y = \dfrac{x}{y^2\cos^2\left(\frac{x}{y}\right)}$

d. $f_x = \dfrac{\sec^2\left(\frac{x}{y}\right)}{y}$ and $f_y = \dfrac{-x}{y^2\cos^2\left(\frac{x}{y}\right)}$

e. None of the above

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7. Use implicit differentiation to calculate the partial derivative $\dfrac{\partial z}{\partial y}$ of the function

$$e^{xy} + \sin(xz) + y = 0$$

a. $\dfrac{\partial z}{\partial y} = \dfrac{xe^{xy}+1}{x\cos(xz)}$

b. $\dfrac{\partial z}{\partial y} = -\dfrac{xe^{xy}+1}{x\cos(xz)}$

c. $\dfrac{\partial z}{\partial y} = -\dfrac{xe^{xy}-1}{x\cos(xz)}$

d. $\dfrac{\partial z}{\partial y} = \dfrac{e^{xy}-1}{x\cos(xz)}$

e. $\dfrac{\partial z}{\partial y} = -\dfrac{e^{xy}+1}{x\sin(xz)}$

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8. Find the directional derivative of $f(x,y) = x^2y - e^xy^3$ at the point $(0,2)$ in the direction of the vector $\langle -2, 3\rangle$.

a. -20

b. -52

c. $-4\sqrt{5}$

d. $-4\sqrt{13}$

e. $\dfrac{-20}{\sqrt{13}}$

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9. Let $f(x, y)$ be defined on an open set D of $\mathbf{R}^2$ which contains the point (a, b). Which of the following statements are true?

P. $\left.\dfrac{\partial f}{\partial x}\right|_{(a,b)}$ is the slope of the tangent line to the curve $f(x, b)$ at the point (a, b).

Q. $\dfrac{\partial^3 f}{\partial x^2 \partial y} = f_{xxy}$

R. $f_{xy} = f_{yx}$

a. P only   b. R only   c. P and Q

d. P and R   e. P, Q, and R

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10. For which value of t does the tangent plane to the graph of the function $z = x^2 y + \dfrac{1}{1+y^2}$ at the point $(1, 1, \frac{3}{2})$ contain the point $(2, 2, t)$?

a. 8.2   b. $1 + \dfrac{7\sqrt{2}}{4}$ c. 4   d. 5   e. None of the above

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11. Calculate the limit:

$$\lim_{(x,y)\to(2,-2)} \frac{x^4 - y^4}{x + y}$$

a. 0   b. 16   c. 32   d. 64   e. The limit does not exist

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12. Let $f(x,y,z) = x^3 + yz^2$, where $x = u^2 + v$, $y = u + v^2$ and $z = uv$. Find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ at $(u,v) = (-1,-1)$.

a. $\left.\dfrac{\partial f}{\partial u}\right|_{(u,v)=(-1,-1)} = 1$ and $\left.\dfrac{\partial f}{\partial v}\right|_{(u,v)=(-1,-1)} = -2$

b. $\left.\dfrac{\partial f}{\partial u}\right|_{(u,v)=(-1,-1)} = -1$ and $\left.\dfrac{\partial f}{\partial v}\right|_{(u,v)=(-1,-1)} = -2$

c. $\left.\dfrac{\partial f}{\partial u}\right|_{(u,v)=(-1,-1)} = -1$ and $\left.\dfrac{\partial f}{\partial v}\right|_{(u,v)=(-1,-1)} = 0$

d. $\left.\dfrac{\partial f}{\partial u}\right|_{(u,v)=(-1,-1)} = 1$ and $\left.\dfrac{\partial f}{\partial v}\right|_{(u,v)=(-1,-1)} = 2$

e. None of the above

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13. If $f(x,y,z) = e^{1-x\cos(y)} + ze^{\frac{-1}{1+y^2}}$, then $\dfrac{\partial f}{\partial x}(1,0,\pi)$ is equal to:

a. -1 b. $\dfrac{-1}{e}$ c. 0 d. $\dfrac{\pi}{e}$ e. 1

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14. Use differentials to approximate the change in $z = 4x - 2y + 3xy$ as (x,y) changes from $(2,-1)$ to $(2.1,-1.1)$.

a. 0.3 b. -0.3 c. 0.5 d. -0.5 e. 0.6

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15. Find the equation of the tangent plane of the one sheet hyperboloid $9x^2 + 4y^2 - z^2 = 25$ at the point $(1,4,0)$.

a. $9x + 16y = 55$ b. $18x + 32y = -146$ c. $18x - 32y = 146$

d. $-18x + 32y = 73$ e. $18x + 32y = 146$

MAC 2313 Exam 2A, Part II Free Response

Name: ________________________________ UF ID #: __________________

Signature: ______________________________ Section #: _________________

<u>SHOW ALL WORK TO RECEIVE FULL CREDIT</u>

1. (a) (2 points) Find the domain of the function $f(x, y) = \ln(9 - 9x^2 - y^2)$.

(b) (2 points) Find the range of $f(x, y)$ (be sure to justify your answer).

(c) (3 points) Identify the level curves of $f(x, y)$.

2. (7 points) If $f(x, y) = \ln(x + y) + 2x^3y^2$, find a vector which points in the direction of no change in the function at the point $(2, -1)$.

3. (7 points) A particle traveling with a position function $\mathbf{r}(t) = \langle 3\cos(\pi t), 3\sin(\pi t), t\rangle$ experiences a force of magnitude $F(x, y, z) = \dfrac{100}{x^2 + y^2 + z^2}$. Find the rate of change of F with respect to time when $t = 1$.

Name: ________________________________ UF ID #: __________________

4. Let $f(x, y, z) = 2\cos x + yz^2$ and $\mathbf{u} = \langle -2, 0, 1\rangle$.

(a) (1 point) Find ∇f.

(b) (3 points) Find $D_{\mathbf{u}} f\left(\frac{\pi}{2}, 1, -1\right)$.

(c) (3 points) At the point $\left(\frac{\pi}{2}, 1, -1\right)$, is there any direction in which the rate of increase of f is 4? Defend your answer.

5. (7 points) Let $f(x,y,z) = e^{\frac{x}{y}} \cos z$. Use the linear approximation at the point $(0,1,0)$ to estimate $e^{\frac{.003}{1.001}} \cos(.002)$.