MAC2313 Test 3 A

Name: ____________________ UF-ID: ________________ Section: ____________

(7 pts) 1. Set up (but do not evaluate) the integral in cylindrical coordinates used to integrate $g(x, y, z) = xyz^2$ over the solid bounded by $z = 1$, $z = 2x^2 + 2y^2 + 8$, and $x^2 + y^2 \leq 4$.

$$\underbrace{\int_0^{2\pi}}_{1pt} \underbrace{\int_0^{2}}_{1pt} \underbrace{\int_1^{2r^2+8}}_{2pts} \underbrace{r^3 \cos\theta \sin\theta \, z^2}_{2pts} \, \underbrace{dz\,dr\,d\theta}_{1pt}$$

(7 pts) 2. Set up (but do not evaluate) the integral obtained by transforming the following $\int_{\pi}^{3\pi/2} \int_0^3 r^2 \sin\theta \, dr \, d\theta$ into Cartesian coordinates.

$$\underbrace{\int_{-3}^{0}}_{2pts} \underbrace{\int_{-\sqrt{9-x^2}}^{0}}_{2pts} \overbrace{y}^{2pts} \, \underbrace{dy\,dx}_{1pt}$$

or

$$\int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0} y \, dx \, dy$$

(7 pts) 3. Set up (but do not evaluate) the integral obtained by transforming the following $\int_0^4 \int_{-\sqrt{16-x^2}}^0 \int_0^{\sqrt{16-x^2-y^2}} (z+y)\, dz\, dy\, dx$ into spherical coordinates.

$$\underbrace{\int_{\frac{3\pi}{2}}^{2\pi}}_{2pts} \underbrace{\int_0^{\frac{\pi}{2}}}_{1pt} \underbrace{\int_0^4}_{1pt} \underbrace{(\rho\cos\phi + \rho\sin\theta\sin\phi)\,\rho^2\sin\phi}_{2pts}\ \underbrace{d\rho\, d\phi\, d\theta}_{1pt}$$

(7 pts) 4. Set up (but do not evaluate) the integral obtained by transforming the following $\int_0^{\pi/2} \int_0^{\pi} \int_0^6 \rho^3 \sin\phi \cancel{\cos\theta} \cos\phi\, d\rho\, d\phi\, d\theta$ into Cartesian coordinates.

$\rho^3 \sin\phi \cos\phi$

$$\underbrace{\int_0^6}_{1pt} \underbrace{\int_0^{\sqrt{36-x^2}}}_{2pts} \underbrace{\int_{-\sqrt{36-x^2-y^2}}^{\sqrt{36-x^2-y^2}}}_{2pts} \overbrace{z}^{1pt}\ \underbrace{dz\, dy\, dx}_{1pt}$$

or

$$\underbrace{\int_0^6}_{1pt} \underbrace{\int_0^{\sqrt{36-y^2}}}_{2pts} \underbrace{\int_{-\sqrt{36-x^2-y^2}}^{\sqrt{36-x^2-y^2}}}_{2pts} \overbrace{z}^{1pt}\ \underbrace{dz\, dx\, dy}_{1pt}$$

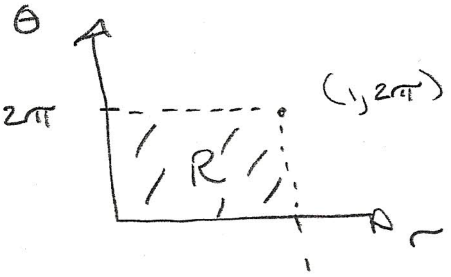
5. Let D be a region in the x, y-plane bounded by the ellipse $\frac{x^2}{7^2} + \frac{y^2}{2^2} = 1$ and define a transformation by $(x, y) = T(r, \theta) = (7r\cos\theta, 2r\sin\theta)$.

(2 pts) a. Calculate $J(r, \theta)$.

$$J(r,\theta) = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \left.\begin{vmatrix} 7\cos\theta & -7r\sin\theta \\ 2\sin\theta & 2r\cos\theta \end{vmatrix}\right\} \text{1pt}$$

$$\left.\begin{aligned} &= 14r\cos^2\theta + 14r\sin^2\theta \\ &= 14r \end{aligned}\right\} \text{1pt}$$

(2 pts) b. Sketch the region in the r, θ-plane which gets mapped into D under the transformation.



(3 pts) c. Convert the integral $\int\int_D 4x^2 + 3y\, dA$ to an integral with respect to the variables r and θ; <u>do not evaluate the integral.</u>

$$\underbrace{\int_0^{2\pi}\int_0^1}_{\text{1pt}} \underbrace{\left[4(7r\cos\theta)^2 + 3(2r\sin\theta)\right](14r)}_{\text{1pt}}\, \underbrace{dr\, d\theta}_{\text{1pt}}$$

or

opposite order