MAC2313 Test 3 A Spring 2015

(5 pts) 1. If $f(x,y) = y\cos(xy)$ and $R = \{\ (x,y) \mid 1 \le x \le 2,\ 0 \le y \le \pi\ \}$ then the value of $\int\int_R f(x,y)\,dA$ is equal to:

A. -2 B. -1 C. 0 D. 1 E. none of the above

$$\int_0^\pi \int_1^2 y\cos(xy)\,dx\,dy = \int_0^\pi \sin(xy)\Big|_1^2\,dy = \int_0^\pi (\sin(2y) - \sin(y))\,dy = -\tfrac{1}{2}\cos(2y) + \cos y\Big|_0^\pi$$
$$= \left[-\tfrac{1}{2} - 1\right] - \left[-\tfrac{1}{2} + 1\right] = -2$$

(5 pts) 2. The integral $\int_0^2 \int_1^2 \int_1^e (4xy)/z\,dz\,dy\,dx$ is equal to:

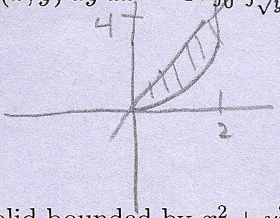
A. 0 B. 6 C. 12 D. 18 E. 24

$$4\left(\int_0^2 x\,dx\right)\left(\int_1^2 y\,dy\right)\left(\int_1^e \frac{1}{z}\,dz\right) = 4\left(\tfrac{1}{2}x^2\Big|_0^2\right)\left(\tfrac{1}{2}y^2\Big|_1^2\right)\left(\ln z\Big|_1^e\right) = 4(2)\left(\tfrac{3}{2}\right)(1) = 12$$

(5 pts) 3. If the order of integration is changed in $\int_0^2 \int_{x^2}^{2x} g(x,y)\,dy\,dx$, the new integral is:

A. $\int_0^2 \int_{y/2}^{\sqrt{y}} g(x,y)\,dy\,dx$ B. $\int_0^4 \int_{y/2}^{\sqrt{y}} g(x,y)\,dy\,dx$ C. $\int_0^2 \int_{\sqrt{y}}^{y/2} g(x,y)\,dy\,dx$ D. $\int_0^4 \int_{\sqrt{y}}^{y/2} g(x,y)\,dy\,dx$

E. none of the above



$$\int_0^4 \int_{y/2}^{\sqrt{y}} g(x,y)\,dx\,dy$$

(5 pts) 4. Let D be the cylindrical solid bounded by $x^2 + y^2 \le d^2$ and $a \le z \le b$, then $\int\int\int_D (9x - 2y)z\,dV = \int_0^{2\pi} \int_0^d \int_a^b (9\cos\theta - 2\sin\theta)zr^{2}\,dz\,dr\,d\theta$.

A. True B. False

(5 pts) 5. The volume between the surfaces $z = x^2 + y^4$ and $z = 10$ over the region $R = \{ (x,y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq x^3 \}$ is given by:

A. $\int_0^1 \int_0^{x^3} x^2 + y^4 - 10\ dy\ dx$ B. $\int_0^1 \int_0^{x^3} x^2 + y^4 + 10\ dx\ dy$ C. $\int_0^1 \int_0^{x^3} 10 - x^2 - y^4\ dx\ dy$

D. $\int_0^1 \int_1^{\sqrt[3]{y}} 10 - x^2 - y^4\ dy\ dx$ $\boxed{\text{E.}}$ $\int_0^1 \int_0^{x^3} 10 - x^2 - y^4\ dy\ dx$

(5 pts) 6. Let D be a bounded solid in R^3, then the volume of D is equal to $\int\int\int_D 1\ dV$.

$\boxed{\text{A. True}}$ B. False

(5 pts) 7. The maximum value of the function $f(x,y,z) = x+y+z$ on the sphere $x^2+y^2+z^2 = 27$ is equal to:

A. 3 B. 6 $\boxed{\text{C. 9}}$ D. 18 E. 27

$\nabla f = \langle 1,1,1 \rangle$ $\nabla g = \langle 2x, 2y, 2z \rangle$

$1 = 2\lambda x$ $\lambda = \frac{1}{2x} = \frac{1}{2y} = \frac{1}{2z}$ $x = y = z$ $3x^2 = 27$

$1 = 2\lambda y$ $x^2 = 9$

$1 = 2\lambda z$

$x = \pm 3$

$x = y = z = 3 \Rightarrow f(3,3,3) = 3+3+3 = 9$

(5 pts) 8. If $f(x,y) = xy^2$ is integrated over the region in the second quadrant bound by $r = 1+\cos\theta$, the correct integral is:

A. $\int_0^{\pi}\int_0^{1+\cos\theta} r^3\cos\theta\sin^2\theta\,dr\,d\theta$

B. $\int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r^3\cos^2\theta\sin^2\theta\,dr\,d\theta$

C. $\int_0^{\pi}\int_0^{1+\cos\theta} r^2\cos^2\theta\sin\theta\,dr\,d\theta$

D. $\int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r^3\cos\theta\sin^2\theta\,dr\,d\theta$

E. none of the above

$\int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r^3\sin^2\theta\cos\theta\, r\,dr\,d\theta$

(5 pts) 9. The integral $\int_{-2}^{2}\int_{x^2}^{4} 12xy\,dy\,dx$ is equal to:

A. 0 B. 64 C. 128 D. 256 E. 512

$\int_{-2}^{2} 6xy^2\Big|_{x^2}^{4}dx = \int_{-2}^{2}(96x - 6x^5)\,dx = 48x^2 - x^6\Big|_{-2}^{2} = 0$ since $48x^2 - x^6$ is even

(5 pts) 10. Let D be the part of the unit ball in the first octant then $\iiint_D xyz\,dV = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho^5\cos\theta\sin\theta\cos\phi\sin^3\phi\,d\rho\,d\phi\,d\theta$.

A. True B. False

$0 \le \rho \le 1$
$0 \le \phi \le \frac{\pi}{2}$
$0 \le \theta \le \frac{\pi}{2}$

$(\rho\sin\phi\cos\theta)(\rho\sin\phi\sin\theta)(\rho\cos\phi)(\rho^2\sin\phi)$
$\rho^5\sin^3\phi\sin\theta\cos\theta\cos\phi$

(5 pts) 11. How many of the following are true?

i. $x = \rho\cos\theta\sin\phi$ ✓

ii. $y = \rho\sin\theta\sin\phi$ ✓

iii. $r^{2} = x^2 + y^2$ X

iv. $\phi = \arctan(z/\sqrt{x^2+y^2+z^2})$ X
arccos (z/ρ)

A. 0 B. 1 $\boxed{\text{C. 2}}$ D. 3 E. 4

(5 pts) 12. Under the transformation $x = r\cos\theta,\ y = r\sin\theta$, the polar rectangle $1 \leq r \leq 2,\ (\pi/2) \leq \theta \leq (3\pi/2)$ gets mapped into which of the following regions?

A. $R = \{\ (x, y) \mid 1 \leq x^2+y^2 \leq 2,\ x \leq 0\ \}$

B. $R = \{\ (x, y) \mid 1 \leq x^2+y^2 \leq 2,\ y \leq 0\ \}$

C. $R = \{\ (x, y) \mid 1 \leq x^2+y^2 \leq 4,\ x \leq 0\ \}$

D. $R = \{\ (x, y) \mid 1 \leq x^2+y^2 \leq 4,\ y \leq 0\ \}$

E. $R = \{\ (x, y) \mid 1 \leq x^2+y^2 \leq 4,\ x, y \leq 0\ \}$

(5 pts) 13. The average value of $g(x, y, z) = 2x + y$ on $D = \{\ (x, y, z) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 1 \leq z \leq 5\ \}$ is equal to:

A. 0 $\boxed{\text{B. 2}}$ C. 4 D. 8 E. 16

$V(D) = 1 \cdot 2 \cdot 4 = 8$

$$\int_0^1\int_0^2\int_1^5 (2x+y)\,dz\,dy\,dx = \int_0^1\int_0^2 4(2x+y)\,dy\,dx = \int_0^1 8xy + 2y^2\Big|_0^2\,dx$$

$$= \int_0^1 (16x+8)\,dx = 8x^2+8x\Big|_0^1 = 16$$

$$\frac{16}{8} = 2$$

Bonus (5 pts) 14. The minimum value of the function $f(x, y) = 2x - 6y$ on the ellipse $x^2 + 3y^2 = 4$ is equal to:

A. 0 B. -2 C. -4 D. -6 $\boxed{\text{E. } -8}$

$\nabla f = \langle 2, -6 \rangle \qquad \nabla g = \langle 2x, 6y \rangle$

$2 = 2\lambda x \quad \Rightarrow \lambda = \frac{1}{x} = \frac{-1}{y} \Rightarrow y = -x$

$-6 = 6\lambda y$

$x^2 + 3(-x)^2 = x^2 + 3x^2 = 4x^2 = 4 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$f(1, -1) = 2 + 6 = 8$

$f(-1, 1) = -2 - 6 = -8$

Name: ______________________ UF-ID: ________________ Section: ____________

(7 pts) 1. Set up (but do not evaluate) the integral in cylindrical coordinates used to integrate $g(x, y, z) = xyz^2$ over the solid bounded by $z = 1$, $z = 2x^2 + 2y^2 + 8$, and $x^2 + y^2 \leq 4$.

$$\underbrace{\int_0^{2\pi}}_{1pt} \underbrace{\int_0^{2}}_{1pt} \underbrace{\int_1^{2r^2+8}}_{2pts} \underbrace{r^3 \cos\theta \sin\theta \, z^2}_{2pts} \underbrace{dz\, dr\, d\theta}_{1pt}$$

(7 pts) 2. Set up (but do not evaluate) the integral obtained by transforming the following $\int_{\pi}^{3\pi/2} \int_0^3 r^2 \sin\theta \, dr \, d\theta$ into Cartesian coordinates.

$$\underbrace{\int_{-3}^{0}}_{2pts} \underbrace{\int_{-\sqrt{9-x^2}}^{0}}_{2pts} \overbrace{y}^{2pts} \underbrace{dy\, dx}_{1pt}$$

or

$$\int_{-3}^{0} \int_{-\sqrt{9-y^2}}^{0} y \, dx \, dy$$

(7 pts) 3. Set up (but do not evaluate) the integral obtained by transforming the following $\int_0^4 \int_{-\sqrt{16-x^2}}^0 \int_0^{\sqrt{16-x^2-y^2}} (z+y)\, dz\, dy\, dx$ into spherical coordinates.

$$\underbrace{\int_{\frac{3\pi}{2}}^{2\pi}}_{2pts} \underbrace{\int_0^{\frac{\pi}{2}}}_{1pt} \underbrace{\int_0^4}_{1pt} \underbrace{(\rho\cos\phi + \rho\sin\theta\sin\phi)\rho^2\sin\phi}_{2pts}\ \underbrace{d\rho\, d\phi\, d\theta}_{1pt}$$

(7 pts) 4. Set up (but do not evaluate) the integral obtained by transforming the following $\int_0^{\pi/2} \int_0^{\pi} \int_0^6 \rho^3 \sin\phi \cancel{\cos\theta} \cos\phi\, d\rho\, d\phi\, d\theta$ into Cartesian coordinates.

$\rho^3 \sin\phi \cos\phi$

$$\underbrace{\int_0^6}_{1pt} \underbrace{\int_0^{\sqrt{36-x^2}}}_{2pts} \underbrace{\int_{-\sqrt{36-x^2-y^2}}^{\sqrt{36-x^2-y^2}}}_{2pts} \overbrace{z}^{1pt}\ \underbrace{dz\, dy\, dx}_{1pt}$$

or

$$\underbrace{\int_0^6}_{1pt} \underbrace{\int_0^{\sqrt{36-y^2}}}_{2pts} \underbrace{\int_{-\sqrt{36-x^2-y^2}}^{\sqrt{36-x^2-y^2}}}_{2pts} \overbrace{z}^{1pt}\ \underbrace{dz\, dx\, dy}_{1pt}$$

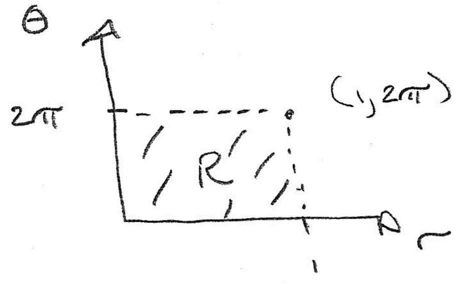
5. Let D be a region in the x,y-plane bounded by the ellipse $\frac{x^2}{7^2} + \frac{y^2}{2^2} = 1$ and define a transformation by $(x,y) = T(r,\theta) = (7r\cos\theta, 2r\sin\theta)$.

(2 pts) a. Calculate $J(r,\theta)$.

$$J(r,\theta) = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \left.\begin{vmatrix} 7\cos\theta & -7r\sin\theta \\ 2\sin\theta & 2r\cos\theta \end{vmatrix}\right\} \text{1pt}$$

$$\left.\begin{aligned} &= 14r\cos^2\theta + 14r\sin^2\theta \\ &= 14r \end{aligned}\right\} \text{1pt}$$

(2 pts) b. Sketch the region in the r,θ-plane which gets mapped into D under the transformation.



(3 pts) c. Convert the integral $\int\int_D 4x^2 + 3y \, dA$ to an integral with respect to the variables r and θ; do not evaluate the integral.

$$\underbrace{\int_0^{2\pi}\int_0^1}_{\text{1pt}} \underbrace{[4(7r\cos\theta)^2 + 3(2r\sin\theta)](14r)}_{\text{1pt}} \underbrace{dr\,d\theta}_{\text{1pt}}$$

or

opposite order