Exam 3 Study Guide

Disclaimer: The following is not intended to be an exhaustive review of everything you should master to do well on Exam 3. Rather, it is meant to supplement your working out of the homework problems and practice exam questions by focusing on terminology and theoretical concepts that might otherwise fall through the cracks. I also am not inside the instructor’s mind and cannot say how close the exam questions will be to this material. These are simply the concepts that jump out to me based on my past experience teaching the class that I thought important enough to highlight.

Terms to Know
Method of Lagrange Multipliers
Lagrange multiplier
iterated integral
average value
polar rectangles
transformation
Jacobian

Theorems to Know
Fubini’s Theorem
Conversions between rectangular, cylindrical, and spherical coordinates
Change of variables

Sample Conceptual Questions

Problem 1. Let \( f(x, y) \) be a function and \( g(x, y) = C \) be a level curve with parameterization \( r(t) \). Then at a critical point of \( f \), \( \nabla f \) and \( \nabla g \) will be _____.

Problem 2. If \( \nabla f = \lambda \nabla g \), we refer to \( \lambda \) as a _____ _____.

Problem 3. The integral \( \iint_D 1 \, dA \) represents the _____ of \( D \), and the integral \( \iiint_W 1 \, dV \) represents the _____ of \( W \).

Problem 4. The integral \( \frac{1}{\text{Area}(D)} \iint_D f(x, y) \, dA \) represents the _____ _____ of \( f \) over \( D \).

Problem 5. If \( T \) is a transformation given by \( x = g(u, v), y = h(u, v) \), then the determinant \( \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} \) is called the _____ of \( T \).

Other Things to Know
Problem 6. When using the method of Lagrange multipliers, what is the system of equations you must solve? When you solve this system, what do you do to find the max and min values?

Problem 7. What does the following integral represent? \( \iint_D f(x, y) \, dA \)

Problem 8. How does “partial integration” work?

Problem 9. How do you switch the order of integration in a double integral, and when might this be useful?

Problem 10. Write down the integral that gives the volume of the solid between the surfaces \( z_1 = f(x, y) \) and \( z_2 = g(x, y) \) over the region \( D \) in the \( xy \)-plane, assuming \( z_1 \geq z_2 \) throughout \( D \).

Problem 11. What is \( dA \) in polar coordinates? What is \( dV \) in cylindrical coordinates? Spherical coordinates?

Problem 12. In what situation would it be advantageous to switch from rectangular to polar or cylindrical coordinates? To spherical coordinates?