NOTE: Be sure to bubble the answers to questions 1–12 on your scantron.

Questions 1-10 are worth 7 points each, 11 is worth 2 points and 12 is worth 3 points.

1. Let $f(x,y)$ be a twice differentiable function with continuous second partial derivatives. Let (a,b) be a critical point of f. Which of the following statements are true?

P. If $D(a,b) > 0$ and $f_{xx} > 0$, then f has a local max at (a,b).

Q. If $D(a,b) > 0$ and $f_{yy} < 0$, then f has a local max at (a,b).

R. If (a,b) is a saddle point of f, then $D(a,b) < 0$.

S. If $D(a,b) = 0$, then (a,b) is not a saddle point and f does not attain min or max at (a,b).

a. P and R only b. Q, R and S only c. Q and R only
d. Q only e. P and S only.

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2. Find the max of $f(x,y) = y^2 + xy - x^2$ on the square $0 \leq x \leq 2, 0 \leq y \leq 2$.

a. 5 b. 4 c. 2 d. 6 e. None of the above.

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3. Find the point on the plane $\frac{x}{2} + \frac{y}{4} + \frac{z}{4} = 1$ closest to the origin in $\mathbb{R}^3$.

a. $(\frac{4}{3}, \frac{2}{3}, \frac{2}{3})$ b. $(\frac{2}{3}, \frac{4}{3}, \frac{4}{3})$ c. $(1,1,1)$ d. $(\frac{2}{3}, \frac{1}{3}, \frac{1}{3})$
e. None of the above.

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4. Evaluate the integral: $\int_0^1 \int_y^1 \frac{\sin(x)}{x} dx dy$.

a. 0 b. $1 - \cos 1$ c. $\cos 1 - 1$ d. π e. None of the above.

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5. Find the average value of $f(x,y) = 3x^2 + 4y$ over the triangular region with vertices $(0,0)$, $(0,2)$ and $(2,0)$.

a. $\frac{7}{3}$ b. $\frac{14}{3}$ c. $\frac{20}{3}$ d. $\frac{28}{3}$ e. $\frac{40}{3}$.

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6. Integrate $f(x,y) = x$ over the region bounded by $y = x^2$ and $y = x + 2$.

a. $\frac{9}{4}$ b. 2 c. 3 d. $\frac{9}{5}$ e. 1.

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7. Express the triple integral in cylindrical coordinates:

$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{x^2+y^2} f(x,y,z)dzdydx.$

a. $\int_0^{\pi} \int_0^1 \int_0^{r^2} f(rcos(\theta), rsin(\theta, z)dzdrd\theta$

b. $\int_0^{\pi} \int_0^1 \int_0^{r^2} f(rcos(\theta), rsin(\theta, z)rdzdrd\theta$

c. $\int_0^{2\pi} \int_0^1 \int_0^{r^2} f(rcos(\theta), rsin(\theta, z)rdzdrd\theta$

d. $\int_0^{2\pi} \int_{-1}^1 \int_0^{r^2} f(rcos(\theta), rsin(\theta, z)rdzdrd\theta$

e. $\int_0^{\pi} \int_{-1}^1 \int_0^{r^2} f(rcos(\theta), rsin(\theta, z)rdzdrd\theta$

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8. Evaluate $\int\int\int_W f(x,y,z)dV$ where $f(x,y,z) = 42(x+y)$ and $W : y \leq z \leq x, 0 \leq y \leq x, 0 \leq x \leq 1$.

a. 5 b. 55 c. 7 d. 42 e. 20

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9. Integrate $f(x,y) = e^{x^2+y^2}$ over $D : x^2 + y^2 \leq 81$.

a. $(e^{81} - 1)\pi$ b. $e^{81}\pi$ c. $(e^{81} - 1)\frac{\pi}{9}$ d. $(e^{9} - 1)\frac{\pi}{81}$
e. $\frac{\pi}{81}$

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10. Which of the following regions describe a solid E that lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$.

a. $E = \{(r, \theta, z) : 0 \leq \theta \leq 2\pi, 0 \leq r \leq 1, 1 - r^2 \leq z \leq 4\}$

b. $E = \{(r, \theta, z) : 0 \leq \theta \leq 2\pi, 0 \leq r \leq 1, 0 \leq z \leq 1 - r^2\}$

c. $E = \{(r, \theta, z) : 0 \leq \theta \leq \pi, 0 \leq r \leq 1, 1 - r^2 \leq z \leq 4\}$

d. $E = \{(r, \theta, z) : 0 \leq \theta \leq \pi, 0 \leq r \leq 1, 0 \leq z \leq 1 - r^2\}$

e. None of the above

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11. Using spherical coordinates to set up the integral to find the volume of the solid ball (sphere) $x^2 + y^2 + z^2 \leq 1$.

a. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^2 \sin(\phi) d\rho d\phi d\theta$

b. $\int_0^{\pi} \int_0^{\pi} \int_0^1 \rho^2 \sin(\phi) d\rho d\phi d\theta$

c. $\int_0^{2\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin(\phi) d\rho d\phi d\theta$

d. $\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \sin(\phi) d\rho d\phi d\theta$

e. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 d\rho d\phi d\theta$

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12. Let D be the parallelogram spanned by vectors $< 2, 3 >$ and $< 4, 1 >$. Apply change of variables formula to the map $\phi(u, v) = (4u + 2v, u + 3v)$ to set up the integral $\int \int_D xy dx dy$ as an integral over $D_0 = [0, 1] \times [0, 1]$.

a. $\int_0^1 \int_0^1 (4u^2 + 6v^2 + 14uv) 10 du dv$

b. $\int_0^1 \int_0^1 (4u^2 + 6v^2) 10 du dv$

c. $\int_0^1 \int_0^1 (4u^2 + 6v^2 + 10uv) du dv$

d. $\int_0^1 \int_0^1 (4u^2 + 6v^2 + 12uv) 10 du dv$

e. None of the above.

MAC 2313 Exam 3A, Part II Free Response

Name: ______________________________ UF ID #: ________________

Signature: ____________________________ Section #: _______________

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10 points) Find the max area of a rectangle inscribed in the ellipse $x^2 + \frac{y^2}{4} = 1$.

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2. (5 points) Evaluate $\int_0^1 \int_{y=x}^1 xe^{y^3}\,dydx$.

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3. (10 points) Evaluate the following integral using polar coordinates: $\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}} dy dx$.

Name: ______________________________ UF ID #: ________________

4.(5 points) Let D be the interior of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Evaluate

$\int\int_D e^{9x^2+4y^2} dxdy$ as an integral over the unit circle (Hint: Use the transformation $x = 2u$, $y = ?$).