Final Exam Study Guide

Disclaimer: The following is not intended to be an exhaustive review of everything you should master to do well on the final exam. Rather, it is meant to supplement your working out of the homework problems and practice exam questions by focusing on terminology and theoretical concepts that might otherwise fall through the cracks. I also am not inside the instructor's mind and cannot say how close the exam questions will be to this material. These are simply the concepts that jump out to me based on my past experience teaching the class that I thought important enough to highlight.

Terms to Know

vector field
radial vector field
conservative vector field (sometimes called gradient field)
potential function
line integral of f over C
line integral of a vector field
closed curve, simple curve
open region, connected region, simply connected region
circulation of the vector field $\mathbf{F}$ around C
surface integral of f over S
flux
surface integral of $\mathbf{F}$ over S
curl of a vector field
irrotational vector field
consistency of orientation between a surface and its boundary curve
divergence of a vector field
incompressible vector field

Theorems to Know: Make sure to know the hypotheses as well as the conclusions!

Cross-Partial Property of Conservative Vector Field
Fundamental Theorem of Conservative Vector Fields
Green's Theorem
Stokes' Theorem
Divergence Theorem

Sample Conceptual Questions

Problem 1. True or false: If $\mathcal{C}$ is a simple closed curve oriented counterclockwise and enclosing a simply connected region D, and $\mathbf{F} = \langle f, g \rangle$ where f and g have continuous first partial derivatives, then $\oint_{\partial D} \mathbf{F}\, d\mathbf{r} = \iint_D (f_x - g_y)\, dA$.

Problem 2. True or false: If D is some region of the plane enclosed by the curve $\mathcal{C}$ satisfying the conditions of Green's Theorem, then $\oint_{\mathcal{C}} (y\tan^2(x))dx + (\tan(x) + e^{y^3})dy$ represents the area of D.

Problem 3. Let $\mathbf{F}$ be a vector field defined on a domain D and $\mathcal{C} : P \to Q$ be a curve lying in D. Which of the following are true?

(1) If $\mathcal{C}_1$ is another path from P to Q, then $\int_{\mathcal{C}_1} \mathbf{F}\, d\mathbf{r} = \int_{\mathcal{C}} \mathbf{F}\, d\mathbf{r}$.

(2) If $\mathcal{C}$ is closed, then $\oint_{\mathcal{C}} \mathbf{F}\, d\mathbf{r} = 0$.

Problem 4. Let $\mathbf{F}$ be a conservative vector field. Which of the following are true?

(1) $\mathrm{curl}(\mathbf{F}) = 0$

(2) $\mathbf{F}$ is irrotational

(3) $\mathbf{F}$ is incompressible

(4) $\mathrm{curl}(\mathbf{F})$ is incompressible

<u>Other Things to Know</u>

Problem 5. How can you tell whether a vector field is conservative?

Problem 6. What do the following integrals represent? $\int_{\mathcal{C}} f(x,y)\, ds$, $\int_{\mathcal{C}} \mathbf{F}\, d\mathbf{r}$, $\iint_{\mathcal{S}} 1\, dS$

Problem 7. Write down the integral that represents the work done by a force field $\mathbf{F}$ in moving an object along the curve $\mathcal{C}$.

Problem 8. When computing line/surface integrals, when do you find the magnitude of a vector, and when do you find a dot product?

Problem 9. If the cross-partial conditions are satisfied by a vector field, what conditions on the domain are required for the vector field to be conservative?

Problem 10. Once you know that a vector field is conservative, how do you find its potential function? (notes method vs. Cyr method)

Problem 11. What is the standard parametrization for a circle of radius r? A cylinder of radius r centered on the z-axis? A sphere of radius R centered at the origin? What are the corresponding normal vectors for the cylinder and sphere?

Problem 12. How do you find a normal vector to a surface? What about in the situation that $z = k(x,y)$?

Problem 13. In what situation would it be advantageous to apply Green's Theorem? Stokes' Theorem? Divergence Theorem?