Problems 1-22 are worth 5 points each.

1. Calculate area of the region enclosed by the line segment connecting $(0,0)$ to $(2,0)$; the part of the parabola $y = 4 - x^2$ from $(2,0)$ to $(0,4)$; and the segment connecting $(0,4)$ to $(0,0)$.

A. 8/3 B. 16/3 C. 32/3 D. 64/3 E. None of the above

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2. Which of the integrals is NOT equal to the area of the region enclosed by the simple closed curve C?

A. $\int_C x\,dy$ B. $\int_C y\,dx$ C. $\int_C \frac{1}{2}x\,dy - \frac{1}{2}y\,dx$ D. $\int_C y\,dx + 2x\,dy$

E. All of the integrals above are equal to the area

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3. Evaluate the line integral

$$\int_C (\cos x - y)\ dx - x\ dy$$

where C is the rectangle oriented counter-clockwise, with the vertices $(1,1)$, $(3,1)$, $(1,4)$ and $(3,4)$.

A. −12 B. −6 C. −4 D. −3 E. None of the above

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4. How many of the following vector fields are unit vector fields (i.e. fields whose vector is of length one for all points.)?

I. $\mathbf{F} = \langle 1, 1, 1 \rangle$

II. $\mathbf{G} = \left\langle \frac{x^2}{x^2+y^2}, \frac{y^2}{x^2+y^2} \right\rangle$

III. $\mathbf{H} = \langle \sin x, \cos y \rangle$

IV. $\mathbf{J} = \langle xy, \sqrt{1 - x^2y^2} \rangle$

A. 0 B. 1 C. 2 D. 3 E. 4

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5. Find a potential function for the vector field $\mathbf{F} = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

A. $V(x, y, z) = x^2y - 5xz - 4zy$ B. $V(x, y, z) = x^2y^2 - 4zy$

C. $V(x, y, z) = x^2y + 5x - 4z^2y$ (D.) $V(x, y, z) = x^2y + 5x - 4zy$

E. None of the above

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6. The vortex field in $\mathbb{R}^2 \setminus \{(0,0)\}$ is the vector field defined by

$$\mathbf{F} = \langle F_1, F_2 \rangle = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$$

Which of the following statements are true about $\mathbf{F}$?

P. $\mathbf{F}$ satisfies the cross partial condition. $\left(\text{i.e. } \dfrac{\partial F_2}{\partial x} = \dfrac{\partial F_1}{\partial y}\right)$.

Q. $\mathbf{F}$ is a conservative vector field. (Hint: Consider the domain.)

R. Let C be the unit circle in counterclockwise direction, parametrized by the vector function $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$. Then $\oint_C \mathbf{F} \cdot \mathbf{T} ds = 0$.

(A.) P only B. P and Q only C. P and R only D. R only E. P, Q and R

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7. A particle moves along a curve $\mathbf{r}(t) = \left\langle t^2, \sin\left(\frac{\pi t}{4}\right), e^{t^2 - 2t} \right\rangle$ where $0 \leq t \leq 2$ in the presence of a force field $\mathbf{F} = \langle 2xyz, x^2z, x^2y \rangle$. Find the work done by the force field in moving the particle from $\langle 0, 0, 1 \rangle$ to $\langle 4, 1, 1 \rangle$.

(A.) 16 B. 4 C. -8

D. -12 E. 18

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8. Find $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$ for the vector field $\mathbf{F} = \langle -y, x-1, -4z+z^3 \rangle$ where C is the top half of the circle with center $(1,0,0)$ and radius 1 on xy-plane oriented counter-clockwise. (Note: C can be parametrized by $\mathbf{r}(t) = \langle 1+\cos t, \sin t, 0 \rangle$ where $0 \le t \le \pi$.)

A. -4π B. π C. 0

D. 4π E. 9π

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9. $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ is zero if

A. $\mathbf{F}$ is tangent to S at every point
B. $\mathbf{F}$ is normal to S at every point
C. the normal component of $\mathbf{F}$ is non-zero at every point
D. S is closed
E. None of the above

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10. Compute $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle y^2, 2, -x \rangle$ and S is the portion of the plane $x+y+z=1$ in the octant $x, y, z \ge 0$ with upward pointing normal.

A. 11 B. 0 C. 1 D. $\frac{11}{12}$ E. $\sqrt{2}$.

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11. Let $\vec{V} = z\vec{k}$ be the velocity field of a fluid (in meters/sec) in $\mathbb{R}^3$. Calculate the flow rate (in cubic meters/sec) through the upper hemisphere ($z \ge 0$) of the sphere $x^2+y^2+z^2=1$.

A. $\frac{\pi}{3}$ B. 2π C. 0 D. $\frac{2\pi}{3}$ E. 1

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12. Let C be a smooth curve in R^3 with parametrization $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ for $\alpha \le t \le \beta$, let $f(x,y,z)$ be a continuous function, and let $\mathbf{F} = \langle f, g, h \rangle$ be a continuous vector field. Which of the following statements are true?

P. $\int_C f ds = \int_\alpha^\beta f(x(t), y(t), z(t)) \cdot \mathbf{r}'(t) dt$

Q. $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C f dx + g dy + h dz$

R. $\int_{-C} \mathbf{F} \cdot \mathbf{T} ds = \int_C \mathbf{F} \cdot \mathbf{T} ds$ (where $-C$ denotes C with the opposite orientation)

S. $\int_C \mathbf{F} \cdot \mathbf{T} ds$ represents the work done by the force field $\mathbf{F}$ in moving an object along the curve C.

A. P and R B. Q and R C. P and S D. Q and S E. P, Q, and S

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13. Evaluate $\oint_C \sin(x)\,dx + z\cos(y)\,dy + \sin(y)\,dz$ where C is the ellipse $4x^2 + 9y^2 = 36$ oriented counterclockwise.

A. 2π B. $2(\pi - 1)$ C. -8

D. 0 E. $2 - 2\pi$

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14. Evaluate the integral $\int\int_S f dS$ where $f(x,y,z) = xy$ and S is the cone $z = 2\sqrt{x^2+y^2}$, $0 \le z \le 4$.

A. 0 B. 1 C. -1 D. 4 E. 5

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15. Calculate $\int\int_S f(x,y,z)dS$ where $f(x,y,z) = z^2$ and S is the part of the plane $x+y+z = 0$ contained in the cylinder $x^2+y^2 = 1$.

A. 0 B. $\frac{\pi}{2}$ C. $\frac{\pi\sqrt{3}}{2}$ D. π E. 2π

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16. If $f(x,y,z) = 4x + z^2$ and C has parametrization $\mathbf{r}(t) = \left\langle \frac{1}{2}t^2, \frac{4}{3}t^{3/2}, 2t \right\rangle$ for $0 \le t \le 2$, then $\int_C f ds$ equals:

A. $\frac{11}{2}$ B. 56 C. $\frac{112}{3}$ D. 80 E. None of the above

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17. If $f(x,y) = 2x$ and C is the part of the parabola $y = x^2$ starting at $(0,0)$ and ending at $(\sqrt{2}, 2)$, then the line integral of f along C equals:

A. $-\frac{1}{12}$ B. $\frac{39}{4}$ C. $\frac{13}{3}$ D. 13 E. None of the above

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18. Evaluate $\int_C 2ydx + xdy$ where C is the top half of the circle $x^2+y^2 = 4$ oriented clockwise and the line segment from $(2,0)$ to $(4,2)$.

A. 0 B. 2π C. 10 D. $10 + 2\pi$ E. $10 - 2\pi$

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19. Calculate the work done by the field $\mathbf{F} = \langle x+y, x-y, y+z \rangle$ in moving an object along the line segment from the point $(1,0,2)$ to the point $(3,-2,6)$.

A. -22 B. -8 C. 0 D. 8 E. 22

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20. Use Green's Theorem to evaluate

$$\int_C (\frac{1}{2}y + e^x)\ dx + (\frac{3}{2}x + \sec y)\ dy.$$

C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. The orientation is chosen to be the counter-clockwise orientation.

A.1/3 B.1/4 C.4 D.3

E. Green's Theorem does not apply because the region is not simply-connected

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21. Find the work done by the force field $\mathbf{F} = -G\hat{\mathbf{j}}$, where G is a constant, in moving an object from $(10, 4)$ to $(2, 1)$.

A.G B.2G C.3G D.5G E. None of the above

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22. Which of the following regions is not a simply connected region?