Practice Exam 2

1) The <u>symmetric group</u> S_n is the collection of permutations on the set $\{1, \dots, n\}$, which is a group under function composition.

2) Does the group S_7 have any elements of order 10?

Yes, $|(12)(34567)| = \operatorname{lcm}(2,5) = 10$.

Does A_7 have any elements of order 10?

No: since there is no room for a 10-cycle, an element of order 10 must be a product of a 2-cycle and a 5-cycle that are disjoint. This element can be written as a product of $1+4=5$ transpositions, so it is an odd permutation and not in A_7.

3) Let $G = (\mathbb{Z}_6, +)$, $H = S_3$. Then G is <u>not</u> isomorphic to H because G is abelian but H is nonabelian (if $G \cong H$ this important group property would be preserved).

4) Let G be a group and $H \leq G$. A <u>left coset</u> of H in G is a subset of G given by $aH = \{ah \mid h \in H\}$ for some $a \in G$.

5) Suppose G is a finite group with $|G| = 33$. Suppose that some element of G has order larger than 12. Prove that G is abelian.

<u>Proof</u> By Lagrange's Thrm, the possible orders of elements of G are $\{1, 3, 11, 33\}$. Since we assumed some element of G has order larger than 12, G must have an element of order 33. Thus, G is cyclic, which implies that G is abelian. ∎

6) Let $\alpha \in S_{15}$ be $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 12 & 1 & 11 & 3 & 8 & 5 & 10 & 6 & 13 & 14 & 7 & 15 & 9 & 4 & 2 \end{bmatrix}$.

Calculate the order of α. Is α an even or odd permutation? Justify your answer.

We have $\alpha = (1, 12, 15, 2)(3, 11, 7, 10, 14, 4)(5, 8, 6)(9, 13)$

$\Rightarrow |\alpha| = \operatorname{lcm}(4, 6, 3, 2) = 12$

Since $\alpha = (1,12)(12,15)(15,2)(3,11)(11,7)(7,10)(10,14)(14,4)(5,8)(8,6)(9,13)$ is a product of 11 transpositions, α is $\boxed{\text{odd}}$.