

ABSTRACT ALGEBRA 1

MAS 4301

TEST 2

October 21, 2016

You may quote standard results (within reason) as long as you make it clear that are doing so and you state them clearly.

In the problems, \mathbf{Z} , resp. \mathbf{Q} , \mathbf{C} , is the set of all integers, resp. rational numbers, complex numbers.

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1. (10 points) Let n be a positive integer. Define what is meant by the symmetric group S_n . ✓
 - 2. (10 points) Does the group S_7 have any elements of order 10? Justify your answer. Does A_7 have any elements of order 10? Justify your answer. ✓
 3. (10 points) Let $G = \mathbf{Z}/6\mathbf{Z}$ under addition, and let $H = S_3$ be the symmetric group of degree 3. Is G isomorphic to H ? Justify your answer. ✓
 4. (10 points) Let G be a group and let H be a subgroup of G . Define what is meant by a left coset of H in G . ✓
 5. (10 points) Suppose G is a finite group of order 33. Suppose that some element of G has order larger than 12. Prove that G is abelian. ✓
 - 6. (10 points) Let $\alpha \in S_{15}$ be the following permutation:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 12 & 1 & 11 & 3 & 8 & 5 & 10 & 6 & 13 & 14 & 7 & 15 & 9 & 4 & 2 \end{bmatrix}. \quad \checkmark$$

Calculate the order of α . Is α an even or an odd permutation? Justify your answer.