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ABSTRACT ALGEBRA 1

MAS 4301

TEST 3

November 18, 2016

You may quote standard results (within reason) as long as you make it clear that are doing so and you state them clearly.

In the problems, $\mathbf{Z}$, resp. $\mathbf{Q}$, $\mathbf{C}$, is the set of all integers, resp. rational numbers, complex numbers.

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1. (10 points) Let $G = \mathbf{Z}/4\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/5\mathbf{Z}$. What is the order of G? Is G cyclic? Justify your answer.

lcm (4,2,5)

2. (10 points) Let $G = \mathbf{Z}/8\mathbf{Z} \oplus \mathbf{Z}/4\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/9\mathbf{Z} \oplus \mathbf{Z}/7\mathbf{Z}$. How many elements of order 2 does G have? Does it have any elements of order 14? Justify your answers.

3. (10 points) Given groups G and H, define what it means to say that f is a *homomorphism* from G to H.

4. (10 points) State the First Isomorphism Theorem for groups.

5. (10 points) Let $G = D_4$ be the dihedral group of order 8. Let

$$f : G \to \mathbf{Z}/4\mathbf{Z}$$

be the map defined by, for $x \in G$,

$$f(x) = \begin{cases} \overline{0} & \text{if } x \text{ is a rotation} \\ \overline{2} & \text{if } x \text{ is not a rotation} \end{cases}$$

has to be onto

Prove that f is a group homomorphism, but not an isomorphism.

Please turn over.

to be isomorphic there are the same # of elements of the same order.

6. (10 points) Let f be as described in 5.

   a) Describe the image of f.

   b) Describe the kernel of f.