(5 pts) 1. Let \( f(x, y, z) \) be a function continuous in \( \mathbb{R}^3 \) and let \( S \) be a surface parameterized by \( \mathbf{r}(u, v) \) with the domain of the parameterization given by \( R \); how many of the following are true? (Assume that the order of the product \( du \ dv \) is consistent with the limits in the double integral.)

a. The vector \( \mathbf{r}_u \times \mathbf{r}_v \) lies in the tangent plane of \( S \) at a given point.

b. \( \int \int_S f(x, y, z) \ dS = \int \int_R f(\mathbf{r}(u, v))|\mathbf{r}_u \times \mathbf{r}_v| \ du \ dv. \)

c. The vectors \( \mathbf{r}_u \) and \( \mathbf{r}_v \) are orthogonal to the tangent plane of \( S \) at a given point.

d. The surface area of \( S \) is given by the integral \( \int \int_R |\mathbf{r}_u \times \mathbf{r}_v| \ du \ dv. \)

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 2. A surface \( S \) is given by the graph of \( z = 3x^2 + 3y^2 - 5, \ \{(x, y) \mid x^2 + y^2 \leq 9\} \) and oriented so that the \( z \) component of the unit normal vector is negative; a unit normal vector to the surface is given by:

A. \((36x^2 + 36y^2 + 1)^{-1/2}(6x, 6y, -1)\)  
B. \((36x^2 + 36y^2 + 1)^{-1/2}(-6x, -6y, -1)\)

C. \((36x^2 + 36y^2 + 1)^{-1/2}(-6x, -6y, 1)\)  
D. \((36x^2 + 36y^2 + 1)^{-1/2}(6x, 6y, 1)\)

E. none of the above

(5 pts) 3. Let \( S \) be the part of the surface \( x + y + z = 1 \) which lies above the triangular region of the \( x, y \)-plane with vertices \((0, 0), (1, 0), \) and \((0, 1)\) and oriented so that the normal vector has a positive \( z \) component. If \( \mathbf{F} = \langle 2, -3, 1 \rangle \), the flux of \( \mathbf{F} \) through \( S \) is given by:

A. 0 B. 1/2 C. -1/2 D. -2 E. 2
Let $C$ be the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$ and having a counterclockwise orientation and let $\mathbf{F} = (y + x^2, e^y - x)$. Which of the following could be used to compute the circulation of $\mathbf{F}$ around $C$?

A. Bubba’s Theorem
B. Clairaut’s Theorem
C. Green’s Theorem
D. the Divergence Theorem
E. the Fundamental Theorem of Line Integrals

The circulation of $\mathbf{F}$ around $C$ in problem 4 has the value:

A. 0  B. $-2$  C. $-4$  D. $-6$  E. $-8$

If $\mathbf{F} = (x^2 e^{3y}, e^{3y} \cos(\pi z), z \ln x)$, then the divergence of $\mathbf{F}$ at the point $(1,0,1)$ is equal to:

A. 0  B. 1  C. 5  D. $e^3$  E. none of the above

If $\mathbf{F} = (2xy^3, xy + zy^2, z^3(x - 2y))$, then the curl of $\mathbf{F}$ at the point $(1,2,-1)$ is equal to:

A. $(−2, 1, −22)$  B. $(2, 1, −22)$  C. $(−2, 1, −22)$  D. $(2, −1, 22)$  E. $(−2, −1, 22)$
(5 pts) 8. If a curve C is given parametrically by \( \mathbf{r}(t) = (3, 2t, t^2) \), \( t : 0 \to \sqrt{3} \) and \( f(x, y, z) = 6xy \), then the value of \( \int_C f(x, y, z) \, ds \) is:

A. 144  B. 146  C. 168  D. 172  E. none of the above

(5 pts) 9. Let \( \mathbf{F} \) be a conservative vector field in \( \mathbb{R}^3 \) and let \( C_1 \) and \( C_2 \) be two simple, smooth curves which share the same initial point \( P \) and terminal point \( Q \); then \( \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \).

A. true  B. false

(5 pts) 10. Let \( \mathbf{F} = (x - y, x + y) \); the circulation of \( \mathbf{F} \) around the unit circle with counterclockwise orientation is equal to:

A. 0  B. 2\( \pi \)  C. \(-2\pi \)  D. 2\( \pi \) + 1  E. \(-2\pi \) - 1

(5 pts) 11. Let \( f(x, y, z) \) be a function continuous in \( \mathbb{R}^3 \) and let \( S \) be a surface given as the graph of the differentiable function \( z = k(x, y) \) with domain \( \mathbb{R} \); how many of the following are true? (Assume that the order of the product \( dy \, dx \) is consistent with the limits in the double integral.)

a. \( \int_S f(x, y, z) \, dS = \int_R f(x, y, k(x, y)) \sqrt{k_x^2 + k_y^2 + 1} \, dy \, dx \).

b. \( \int_S [f(x, y, z)]^2 \, dS = \int_R [f(x, y, k(x, y))]^2 \sqrt{k_x^2 + k_y^2 + 1} \, dy \, dx \).

c. If \( \mathbf{r}(x, y) = (x, y, k(x, y)) \), then \( \mathbf{r}_x \times \mathbf{r}_y \) is normal to the surface.

d. The vector \( (k_x, k_y, 1) \) is orthogonal to the tangent plane of \( S \) at a given point.

A. 0  B. 1  C. 2  D. 3  E. 4
12. Let $S$ be the unit sphere centered at the origin and let $\mathbf{F} = \langle -2x, 3y, 5z \rangle$, then the flux across the surface is given by:

A. 0  B. $6\pi$  C. $8\pi$  D. $12\pi$  E. $16\pi$

13. Let $S$ be the part of the cylinder of radius 5 centered on the $z$ axis with $x \leq 0$ and $|z| \leq 1$; then $S$ is given parametrically by $\mathbf{r}(\theta, z) = \langle 5 \cos \theta, 5 \sin \theta, z \rangle$ with domain $R = \{ (\theta, z) \ | \ \pi \leq \theta \leq 2\pi, -1 \leq z \leq 1 \}$.

A. true  B. false

14. If $\text{curl} \mathbf{F} = 0$, then $\mathbf{F}$ is called irrotational.

A. true  B. false

15. Let $\mathbf{F}$ be any continuous vector field given by $\mathbf{F} = \nabla \phi(x, y)$; let $C$ be any simple, smooth, open, oriented curve in $\mathbb{R}^2$, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 0.$$

A. true  B. false

16. If $\mathbf{F} = \langle y, x + z^2, 2zy \rangle$ and $C$ is given parametrically by $\mathbf{r}(t) = \langle 2, \cos^3 t, \sin^3 t \rangle$, $t : 0 \to \pi/2$, then what is the value of $\int_C \mathbf{F} \cdot \mathbf{T} \, ds$?

A. $-3$  B. $-2$  C. $-1$  D. 0  E. 4
(5 pts) 17. If \( f(x, y, z) = z(x^2 + y^2) \) and \( S \) the part of the cone \( z = 2\sqrt{x^2 + y^2} \), \( 0 \leq z \leq 4 \), then \( \int_S f(x, y, z) \, dS \) is equal to which of the following:

A. \( 2\sqrt{5} \int_0^{2\pi} \int_0^2 r^4 \, dr \, d\theta \)  
B. \( 2\sqrt{5} \int_0^{2\pi} \int_0^2 r^3 \, dr \, d\theta \)  
C. \( 2\sqrt{5} \int_0^{2\pi} \int_0^2 r^5 \, dr \, d\theta \)  
D. \( 5\sqrt{2} \int_0^{2\pi} \int_0^4 r^4 \, dr \, d\theta \)  
E. none of the above

(5 pts) 18. If \( \mathbf{F}(x, y, z) = \langle x^2, 3y + x, 2y \rangle \) and \( S \) is a surface given by \( \mathbf{r}(u, v) = \langle 3u, 2u \sin v, 0 \rangle \), \( 0 \leq u \leq 1 \), \( 0 \leq v \leq \pi/2 \) and oriented so that the \( z \) component of the normal vector is positive; then \( \int_S \mathbf{F}(x, y, z) \cdot \mathbf{n} \, dS \) is equal to:

A. 0  
B. 2  
C. 4  
D. 6  
E. 8

(5 pts) 19. Let \( \mathbf{F} \) be a constant vector field and let \( S \) be a smooth, closed surface; then the statement \( \int_S \mathbf{F} \cdot \mathbf{n} \, dS = 0 \) is true:

A. sometimes  
B. always  
C. never

(5 pts) 20. Assuming the limits for the integrals are consistent with the given properties, which of the following are true:

I. Stoke’s Theorem is given by \( \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = \int_C \mathbf{F} \cdot \mathbf{T} \, ds \).

II. If \( \mathbf{F} \) is constant and \( S \) is a smooth, bounded, closed surface, then \( \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = 0 \).

III. If \( \mathbf{F} \) is constant and \( C \) is a smooth, simple closed curve, then \( \int_C \mathbf{F} \cdot d\mathbf{r} = 0 \).

IV. The Divergence Theorem is given by \( \int_S \nabla \cdot \mathbf{F} \, dS = \int_D \mathbf{F} \cdot \mathbf{n} \, dV \).

A. only I  
B. only III  
C. only I and II  
D. only I, II, and IV  
E. I, II, and III
Bonus (5 pts) 21. Let $S$ be the part of the unit sphere with $z \geq 0$ oriented so that the $z$ component of the unit normal vector is positive; if $\mathbf{F} = \langle -y/2, x/2, z^2(x+y) \rangle$, then $\int \int_S (\nabla \times \mathbf{F}) \cdot \hat{n} \, dS$ is equal to: (Don’t compute this integral directly.)

A. 0  B. $-\pi$  C. $\pi$  D. $-2\pi$  E. $2\pi$

Bonus (5 pts) 22. Let $\mathbf{F}$ be a radial vector field given by $\frac{\mathbf{r}}{|\mathbf{r}|^5}$; a potential function for $\mathbf{F}$ is:

A. $(1/3)(x^2 + y^2 + z^2)^{-3/2}$  B. $(2/3)(x^2 + y^2 + z^2)^{-3/2}$  C. $(1/7)(x^2 + y^2 + z^2)^{-7/2}$

D. $(2/7)(x^2 + y^2 + z^2)^{-7/2}$  E. none of the above