MAC2313 Final A

(5 pts) 1. Let $f(x, y, z)$ be a function continuous in R^3 and let S be a surface parameterized by $\mathbf{r}(\mathrm{u}, \mathrm{v})$ with the domain of the parameterization given by R; how many of the following are true? (Assume that the order of the product $du\ dv$ is consistent with the limits in the double integral.)

a. The vector $\mathbf{r_u} \times \mathbf{r_v}$ lies in the tangent plane of S at a given point.

b. $\int\int_{\mathrm{S}} \mathrm{f(x, y, z)\ dS} = \int\int_{\mathrm{R}} \mathrm{f}(\mathbf{r}(\mathrm{u}, \mathrm{v}))|\mathbf{r_u} \times \mathbf{r_v}|\ \mathrm{du\ dv}$.

c. The vectors $\mathbf{r_u}$ and $\mathbf{r_v}$ are orthogonal to the tangent plane of S at a given point.

d. The surface area of S is given by the integral $\int\int_R |\mathbf{r_u} \times \mathbf{r_v}|\ \mathrm{du\ dv}$.

A. 0   B. 1   C. 2   D. 3   E. 4

(5 pts) 2. A surface S is given by the graph of $z = 3x^2 + 3y^2 - 5,\ \{(x, y) \mid x^2 + y^2 \leq 9\}$ and oriented so that the z component of the unit normal vector is negative; a unit normal vector to the surface is given by:

A. $(36x^2 + 36y^2 + 1)^{-1/2} \langle 6x, 6y, -1\rangle$ B. $(36x^2 + 36y^2 + 1)^{-1/2} \langle -6x, -6y, -1\rangle$

C. $(36x^2 + 36y^2 + 1)^{-1/2} \langle -6x, -6y, 1\rangle$ D. $(36x^2 + 36y^2 + 1)^{-1/2} \langle 6x, 6y, 1\rangle$

E. none of the above

(5 pts) 3. Let S be the part of the surface $x + y + z = 1$ which lies above the triangular region of the x, y - plane with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$ and oriented so that the normal vector has a positive z component. If $\mathbf{F} = \langle 2, -3, 1\rangle$, the flux of $\mathbf{F}$ through S is given by:

A. 0   B. 1/2   C. $-1/2$ D. -2 E. 2

(5 pts) 4. Let C be the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ and having a counter-clockwise orientation and let $\mathbf{F} = \langle \mathrm{y} + \mathrm{x}^2, \mathrm{e}^{\mathrm{y}} - \mathrm{x} \rangle$. Which of the following could be used to compute the circulation of $\mathbf{F}$ around C?

A. Bubba's Theorem

B. Clairaut's Theorem

C. Green's Theorem

D. the Divergence Theorem

E. the Fundamental Theorem of Line Integrals

(5 pts) 5. The circulation of $\mathbf{F}$ around C in problem 4 has the value:

A. 0 B. -2 C. -4 D. -6 E. -8

(5 pts) 6. If $\mathbf{F} = \langle \mathrm{x}^2\mathrm{e}^{3\mathrm{y}}, \mathrm{e}^{3\mathrm{y}}\cos(\pi \mathrm{z}), \mathrm{z}\ln \mathrm{x} \rangle$, then the divergence of $\mathbf{F}$ at the point $(1,0,1)$ is equal to:

A. 0 B. 1 C. 5 D. e^3 E. none of the above

(5 pts) 7. If $\mathbf{F} = \langle 2\mathrm{xy}^3, \mathrm{xy} + \mathrm{zy}^2, \mathrm{z}^3(\mathrm{x} - 2\mathrm{y}) \rangle$, then the curl of $\mathbf{F}$ at the point $(1,2,-1)$ is equal to:

A. $\langle -2, 1, -22 \rangle$ B. $\langle 2, 1, -22 \rangle$ C. $\langle -2, 1, -22 \rangle$ D. $\langle 2, -1, 22 \rangle$ E. $\langle -2, -1, 22 \rangle$

(5 pts) 8. If a curve C is given parametrically by $\mathbf{r}(\mathrm{t}) = \langle 3, 2\mathrm{t}, \mathrm{t}^2 \rangle$, $\mathrm{t} : 0 \to \sqrt{3}$ and $f(x, y, z) = 6xy$, then the value of $\int_C f(x, y, z)\, ds$ is:

A. 144 B. 146 C. 168 D. 172 E. none of the above

(5 pts) 9. Let $\mathbf{F}$ be a conservative vector field in R^3 and let C_1 and C_2 be two simple, smooth curves which share the same initial point P and terminal point Q; then $\int_{C_1} \mathbf{F} \cdot \mathrm{d}\mathbf{r} = \int_{\mathrm{C}_2} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

A. true B. false

(5 pts) 10. Let $\mathbf{F} = \langle \mathrm{x} - \mathrm{y}, \mathrm{x} + \mathrm{y} \rangle$; the circulation of $\mathbf{F}$ around the unit circle with counterclockwise orientation is equal to:

A. 0 B. 2π C. -2π D. $2\pi + 1$ E. $-2\pi - 1$

(5 pts) 11. Let $f(x, y, z)$ be a function continuous in R^3 and let S be a surface given as the graph of the differentiable function $z = k(x, y)$ with domain R; how many of the following are true? (Assume that the order of the product $dy\, dx$ is consistent with the limits in the double integral.)

a. $\int\int_{\mathrm{S}} \mathrm{f(x, y, z)}\ \mathrm{dS} = \int\int_{\mathrm{R}} \mathrm{f(x, y, k(x, y))} \sqrt{\mathrm{k_x^2 + k_y^2 + 1}}\ \mathrm{dy\ dx}$.

b. $\int\int_{\mathrm{S}} [\mathrm{f(x, y, z)}]^2\ \mathrm{dS} = \int\int_{\mathrm{R}} [\mathrm{f(x, y, k(x, y))}]^2 \sqrt{\mathrm{k_x^2 + k_y^2 + 1}}\ \mathrm{dy\ dx}$.

c. If $\mathbf{r}(\mathrm{x, y}) = \langle \mathrm{x, y, k(x, y)} \rangle$, then $\mathbf{r_x} \times \mathbf{r_y}$ is normal to the surface.

d. The vector $\langle k_x, k_y, 1 \rangle$ is orthogonal to the tangent plane of S at a given point.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 12. Let S be the unit sphere centered at the origin and let $\mathbf{F} = \langle -2\mathrm{x}, 3\mathrm{y}, 5\mathrm{z} \rangle$, then the flux across the surface is given by:

A. 0 B. 6π C. 8π D. 12π E. 16π

(5 pts) 13. Let S be the part of the cylinder of radius 5 centered on the z axis with $x \leq 0$ and $|z| \leq 1$; then S is given parametrically by $\mathbf{r}(\theta, \mathrm{z}) = \langle 5\cos\theta, 5\sin\theta, \mathrm{z} \rangle$ with domain $R = \{\ (\theta, z) \mid \pi \leq \theta \leq 2\pi,\ -1 \leq z \leq 1\ \}$.

A. true B. false

(5 pts) 14. If curl $\mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is called irrotational.

A. true B. false

(5 pts) 15. Let $\mathbf{F}$ be any continuous vector field given by $\mathbf{F} = \nabla\phi(\mathrm{x}, \mathrm{y})$; let C be any simple, smooth, open, oriented curve in R^2, then

$$\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r} = 0.$$

A. true B. false

(5 pts) 16. If $\mathbf{F} = \langle \mathrm{y}, \mathrm{x} + \mathrm{z}^2, 2\mathrm{zy} \rangle$ and C is given parametrically by $\mathbf{r}(\mathrm{t}) = \left\langle 2, \cos^3 \mathrm{t}, \sin^3 \mathrm{t} \right\rangle$, $t : 0 \to \pi/2$, then what is the value of $\int_C \mathbf{F} \cdot \mathbf{T}\ \mathrm{ds}$?

A. -3 B. -2 C. -1 D. 0 E. 4

(5 pts) 17. If $f(x,y,z) = z(x^2+y^2)$ and S the part of the cone $z = 2\sqrt{x^2+y^2}$, $0 \le z \le 4$, then $\int\int_S \ f(x,y,z)\ dS$ is equal to which of the following:

A. $2\sqrt{5}\int_0^{2\pi}\int_0^2 \ r^4\ dr\ d\theta$ B. $2\sqrt{5}\int_0^{2\pi}\int_0^2 \ r^3\ dr\ d\theta$ C. $2\sqrt{5}\int_0^{2\pi}\int_0^2 \ r^5\ dr\ d\theta$

D. $5\sqrt{2}\int_0^{2\pi}\int_0^4 \ r^4\ dr\ d\theta$ E. none of the above

(5 pts) 18. If $\mathbf{F}(\mathrm{x,y,z}) = \langle \mathrm{x}^2, 3\mathrm{y}+\mathrm{x}, 2\mathrm{y}\rangle$ and S is a surface given by $\mathbf{r}(\mathrm{u,v}) = \langle 3\mathrm{u}, 2\mathrm{u}\sin\mathrm{v}, 0\rangle$, $0 \le u \le 1$, $0 \le v \le \pi/2$ and oriented so that the z component of the normal vector is positive; then $\int\int_S \ \mathbf{F}(\mathrm{x,y,z}) \cdot \hat{\mathbf{n}}\ \mathrm{dS}$ is equal to:

A. 0 B. 2 C. 4 D. 6 E. 8

(5 pts) 19. Let $\mathbf{F}$ be a constant vector field and let S be a smooth, closed surface; then the statement $\int\int_S \ \mathbf{F}\cdot\hat{\mathbf{n}}\ \mathrm{dS} = 0$ is true:

A. sometimes B. always C. never

(5 pts) 20. Assuming the limits for the integrals are consistent with the given properties, which of the following are true:

I. Stoke's Theorem is given by $\int\int_S (\nabla\times\mathbf{F})\cdot\ \hat{\mathbf{n}}\ \mathrm{dS} = \int_C \mathbf{F}\cdot\mathbf{T}\ \mathrm{ds}$.

II. If $\mathbf{F}$ is constant and S is a smooth, bounded, closed surface, then $\int\int_S (\nabla\times\mathbf{F})\cdot\hat{\mathbf{n}}\ \mathrm{dS} = 0$.

III. If $\mathbf{F}$ is constant and C is a smooth, simple closed curve, then $\int_C \ \mathbf{F}\cdot\mathrm{d}\mathbf{r} = 0$.

IV. The Divergence Theorem is given by $\int\int_S \nabla\cdot\mathbf{F}\ \mathrm{dS} = \int\int\int_{\mathrm{D}} \ \mathbf{F}\cdot\hat{\mathbf{n}}\ \mathrm{dV}$.

A. only I B. only III C. only I and II D. only I, II, and IV E. I, II, and III

Bonus (5 pts) 21. Let S be the part of the unit sphere with $z \geq 0$ oriented so that the z component of the unit normal vector is positive; if $\mathbf{F} = \langle -\mathrm{y}/2, \mathrm{x}/2, \mathrm{z}^2(\mathrm{x}+\mathrm{y})\rangle$, then $\int\int_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}}\, \mathrm{dS}$ is equal to: (Don't compute this integral directly.)

A. 0 B. $-\pi$ C. π D. -2π E. 2π

Bonus (5 pts) 22. Let $\mathbf{F}$ be a radial vector field given by $\dfrac{\mathbf{r}}{|\mathbf{r}|^5}$; a potential function for $\mathbf{F}$ is:

A. $(1/3)(x^2+y^2+z^2)^{-3/2}$ B. $(2/3)(x^2+y^2+z^2)^{-3/2}$ C. $(1/7)(x^2+y^2+z^2)^{-7/2}$

D. $(2/7)(x^2+y^2+z^2)^{-7/2}$ E. none of the above