(5 pts) 1. How many of the following are necessarily true?

i. The vector field $\mathbf{F} = \langle -2x + 3y, 3x - 5y \rangle$ is conservative.

ii. The vector field $\mathbf{F} = 5(x^2 + y^2)^{-3/2} \langle x, y \rangle$ is radial.

iii. All constant vector fields in $\mathbb{R}^3$ are conservative.

iv. Gravitational fields in $\mathbb{R}^3$ are conservative and radial.

A. 0  B. 1  C. 2  D. 3  E. 4

(5 pts) 2. How many of the following are necessarily true?

i. The potential function for a conservative vector field is unique.

ii. The divergence of a constant vector field in $\mathbb{R}^3$ is equal to the zero vector.

iii. The divergence of a conservative vector field in $\mathbb{R}^3$ is a constant.

iv. The curl of a conservative vector field in $\mathbb{R}^3$ is equal to the zero vector.

A. 0  B. 1  C. 2  D. 3  E. 4

(5 pts) 3. Let $C$ be the curve given parametrically by $\mathbf{r}(t) = \langle t + 3, 4 - 2t \rangle$, $t : 1 \rightarrow 2$; if $f(x, y) = 2x + 4y$, the value of $\int_C f(x, y) \, ds$ is:

A. 0  B. $13\sqrt{5}$  C. $4\sqrt{5}$  D. $26\sqrt{5}$  E. $22\sqrt{5}$
(5 pts) 4. Let \( C \) be the curve given parametrically by \( \mathbf{r}(t) = (3t, 1 - 2t^2, 4), \ t : 0 \to 2 \); the value of \( \int_C y \, dx + x^2 \, dy + z \, dz \) is:

A. 0  
B. -40  
C. -86  
D. -154  
E. -210

(5 pts) 5. Let \( C \) be the upper half of the unit circle oriented clockwise and let \( \mathbf{F} = (2y + 1, -2x - 1) \), the value of \( \int_C \mathbf{F} \cdot d\mathbf{r} \) is:

A. 0  
B. -2 - \pi  
C. 2 + 2\pi  
D. -2 - 2\pi  
E. 2 + \pi

(5 pts) 6. Let \( D = \{ (x, y) \mid |x| < 1, |y| < 2 \} \); how many of the following are true?

i. \( D \) is simply connected.

ii. \( D \) is connected.

iii. \( D \) is open.

iv. The boundary curve(s) for \( D \) is closed but not simple.

A. 0  
B. 1  
C. 2  
D. 3  
E. 4

(5 pts) 7. Let \( C \) be a simple, smooth curve with initial point \((1, 0, 1)\) and terminal point \((0, 0, -1)\); if \( \mathbf{F} = (6yz^2e^{2xy}, 6xz^2e^{2xy}, 6ze^{2xy} + 1) \), the value of \( \int_C \mathbf{F} \cdot d\mathbf{r} \) is:

A. 0  
B. -2  
C. -4  
D. 3  
E. none of the above
8. How many of the following are necessarily true?

i. The line integral of a conservative vector field is independent of the path connecting an initial point $P$ to a terminal point $Q$.

ii. The curl of a constant vector field in $R^3$ is equal to the scalar zero.

iii. A vector field $\mathbf{F} = \langle f(x, y), g(x, y) \rangle$ is conservative if $f_x = g_y$.

iv. The circulation of a conservative vector field along a smooth, oriented curve is equal to zero.

A. 0  B. 1  C. 2  D. 3  E. 4

9. The part of the unit sphere with $x \leq 0$ and $z \leq 0$ is parameterized by $\mathbf{r}(\theta, \phi) = \langle \cos \theta \sin \phi, \sin \theta \sin \phi, \cos \phi \rangle$; the domain for this parameterization is equal to:

A. $D = \{(\theta, \phi) \mid 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi \}$  
B. $D = \{(\theta, \phi) \mid \pi/2 \leq \theta \leq \pi, 0 \leq \phi \leq \pi/2 \}$  
C. $D = \{(\theta, \phi) \mid \pi \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi/2 \}$  
D. $D = \{(\theta, \phi) \mid \pi/2 \leq \theta \leq 3\pi/2, \pi/2 \leq \phi \leq \pi \}$  
E. $D = \{(\theta, \phi) \mid \pi \leq \theta \leq 2\pi, \pi/2 \leq \phi \leq \pi \}$

10. Let $f(x, y, z) = z + y^2$ and let $S$ be the surface parameterized by $\mathbf{r}(u, v) = \langle 2u, -3v, u + v \rangle$ with $0 \leq u \leq 1, 0 \leq v \leq 2$. The integral $\int\int_S f(x, y, z) \, dS$ is equal to:

A. 0  B. 160  C. 189  D. 84  E. 62
11. Let $S$ be the surface parameterized by $\mathbf{r}(u, v) = \langle 2v, u^2 + v^2, u \rangle$ with $0 \leq u \leq 2$, $0 \leq v \leq 2$. A vector normal to the tangent plane of the surface at $u = 1$, $v = 1$ is:

A. $\langle -2, 2, -4 \rangle$  
B. $\langle 1, -2, 2 \rangle$  
C. $\langle 1, 2, -4 \rangle$  
D. $\langle -2, 1, -4 \rangle$  
E. $\langle 2, 2, -4 \rangle$

12. Let $f(x, y, z) = xye^z$ and let $S$ be the part of the paraboloid $z = x^2 + y^2 + 8$ where $x^2 + y^2 \leq 5$. The integral $\iint_S f(x, y, z) \, dS$ is equal to:

A. $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5} - x^2}^{\sqrt{5} - x^2} xye^2 \, dy \, dx$  
B. $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5} - x^2}^{\sqrt{5} - x^2} xye^2 + y^2 + 8 \, dy \, dx$

C. $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5} - x^2}^{\sqrt{5} - x^2} xye^2 + y^2 + 8 \sqrt{4x^2 + 4y^2 + 1} \, dy \, dx$  
D. $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5} - x^2}^{\sqrt{5} - x^2} xye^2 + y^2 + 8 \sqrt{2x + 2y + 1} \, dy \, dx$

E. none of the above

13. Let $\mathbf{F} = \langle z, x, y \rangle$ and let $S$ be the surface parameterized by $\mathbf{r}(u, v) = \langle \cos u, \sin u, 2v + 3 \rangle$ with domain $R = \{(u, v) \mid 0 \leq u \leq 2\pi, 0 \leq v \leq 2\}$ and oriented so that normal vectors to the surface are pointing away from the $z$-axis; the integral $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$ is given by:

A. $\int_0^{2\pi} \int_0^2 6v \cos u + 6 \sin u \, dv \, du$  
B. $\int_0^{2\pi} \int_0^2 6 \sin u + 6v + v \sin u \cos u \, dv \, du$

C. $\int_0^{2\pi} \int_0^2 4v \cos u + 6 \cos u + 2 \sin u \cos u \, dv \, du$  
D. $\int_0^{2\pi} \int_0^2 6 \cos u + \sin u \cos u \, dv \, du$

E. $\int_0^{2\pi} \int_0^2 12v \cos u \sin u \, dv \, du$
(5 pts) 14. Let $F = \langle x, y, z \rangle$ and let $S$ be the part of the plane $x + y + z = 1$ defined above the triangular region in the $x, y$-plane with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$ and oriented so that the $z$ component of the normal vector is positive; the flux of $F$ across $S$ is given by:

A. $\int_0^1 \int_0^{1-x} 1 \, dy \, dx$
B. $\int_0^1 \int_0^{1-x+1} 1 \, dy \, dx$
C. $\int_0^1 \int_0^{1-x+1} x + y \, dy \, dx$
D. $\int_0^1 \int_0^{1-x+1} 1 - x - y \, dy \, dx$
E. none of the above

(5 pts) 15. Let $C$ be the closed curve in the $x, y$-plane given by the triangle with vertices $(-1, 0)$, $(0, 2)$, and $(1, 0)$ and oriented counter-clockwise. If $F = \langle 4y - 1, 3 - 2x \rangle$, then the circulation of $F$ around $C$ is equal to:

A. 0
B. -6
C. 12
D. -12
E. 6

(5 pts) 16. Let $F = \langle 2xy + 4z^2, x^2 + 2z, 2y + 8xz \rangle$; the divergence of $F$ at the point $(x, y, z) = (1, 1, 1)$ is equal to:

A. 0
B. 4
C. 10
D. 12
E. 6

(5 pts) Bonus 17. The vector field in problem 16 is conservative.

A. True
B. False
18. Let \( F = \langle xz, xy, yz \rangle \); the curl of \( F \) at the point \((x, y, z) = (0, 0, 0)\) is equal to:

A. \( \langle -1, -1, -1 \rangle \)  
B. \( \langle 1, 1, 1 \rangle \)  
C. \( \langle 1, -1, 1 \rangle \)  
D. \( \langle 0, 0, 0 \rangle \)  
E. \( \langle 1, 1, 1 \rangle \)

19. The vector field in problem 18 is conservative.

A. True  
B. False

20. Let \( S \) be a closed surface bounded by a cylinder of radius one centered on the \( z \)-axis, \( z = 0 \), and \( z = 2 \) and let \( F = \langle -6y, 2x, z^2 \rangle \), then the flux across the surface is equal to:

A. 0  
B. \( \pi \)  
C. 4\( \pi \)  
D. 3\( \pi \)  
E. 2\( \pi \)

21. Let \( S \) be the part of the paraboloid \( z = 9 - x^2 - y^2 \) with \( z \geq 0 \) and oriented so that the \( z \) component of the normal vector is positive; if \( F = \langle -y + z, x, e^{x^2+y^2} \rangle \), then the integral \( \int_S (\nabla \times F) \cdot \hat{n} \, dS \) is equal to:

A. 0  
B. 6\( \pi \)  
C. 3\( \pi \)  
D. 2\( \pi \)  
E. 18\( \pi \)

22. Which of the following are true:

I. The Divergence Theorem is given by \( \int \int \int_D F \cdot \hat{n} \, dV = \int \int_S \nabla \cdot F \, dS \).

II. Stoke’s Theorem is given by \( \int_C F \cdot T \, ds = \int \int_S (\nabla \times F) \cdot \hat{n} \, dS \).

III. If \( F \) is conservative and \( S \) is a smooth bounded surface, then \( \int \int_S (\nabla \times F) \cdot \hat{n} \, dS = 0 \).

IV. The flux of a constant vector field out of a solid is always greater than zero. (Hint: consider the Divergence Theorem)

A. only II and III  
B. only I and III  
C. only I, II, and IV  
D. only I, II, and III  
E. I, II, III, and IV