MAC2313 Final A

(5 pts) 1. How many of the following are necessarily true?

i. The vector field $\mathbf{F} = \langle -2\mathrm{x} + 3\mathrm{y}, 3\mathrm{x} - 5\mathrm{y} \rangle$ is conservative.

ii. The vector field $\mathbf{F} = 5(\mathrm{x}^2 + \mathrm{y}^2)^{-3/2} \langle \mathrm{x}, \mathrm{y} \rangle$ is radial.

iii. All constant vector fields in R^3 are conservative.

iv. Gravitational fields in R^3 are conservative and radial.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 2. How many of the following are necessarily true?

i. The potential function for a conservative vector field is unique.

ii. The divergence of a constant vector field in R^3 is equal to the zero vector.

iii. The divergence of a conservative vector field in R^3 is a constant.

iv. The curl of a conservative vector field in R^3 is equal to the zero vector.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 3. Let C be the curve given parametrically by $\mathbf{r}(\mathrm{t}) = \langle \mathrm{t} + 3, 4 - 2\mathrm{t} \rangle$, $\mathrm{t} : 1 \to 2$; if $f(x,y) = 2x + 4y$, the value of $\int_C f(x,y)\, ds$ is:

A. 0 B. $13\sqrt{5}$ C. $4\sqrt{5}$ D. $26\sqrt{5}$ E. $22\sqrt{5}$

(5 pts) 4. Let C be the curve given parametrically by $\mathbf{r}(\mathrm{t}) = \langle 3\mathrm{t}, 1 - 2\mathrm{t}^2, 4 \rangle$, $\mathrm{t} : 0 \to 2$; the value of $\int_C y\ dx + x^2\ dy + z\ dz$ is:

A. 0 B. -40 C. -86 D. -154 E. -210

(5 pts) 5. Let C be the upper half of the unit circle oriented clockwise and let $\mathbf{F} = \langle 2\mathrm{y} + 1, -2\mathrm{x} - 1 \rangle$, the value of $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ is:

A. 0 B. $-2 - \pi$ C. $2 + 2\pi$ D. $-2 - 2\pi$ E. $2 + \pi$

(5 pts) 6. Let $D = \{\ (x, y) \mid |x| < 1,\ |y| < 2\}$; how many of the following are true?

i. D is simply connected.

ii. D is connected.

iii. D is open.

iv. The boundary curve(s) for D is closed but not simple.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 7. Let C be a simple, smooth curve with initial point $(1, 0, 1)$ and terminal point $(0, 0, -1)$; if $\mathbf{F} = \langle 6\mathrm{yz}^2\mathrm{e}^{2\mathrm{xy}}, 6\mathrm{xz}^2\mathrm{e}^{2\mathrm{xy}}, 6\mathrm{ze}^{2\mathrm{xy}} + 1 \rangle$, the value of $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ is:

A. 0 B. -2 C. -4 D. 3 E. none of the above

(5 pts) 8. How many of the following are necessarily true?

i. The line integral of a conservative vector field is independent of the path connecting an initial point P to a terminal point Q.

ii. The curl of a constant vector field in R^3 is equal to the scalar zero.

iii. A vector field $\mathbf{F} = \langle \mathrm{f(x, y), g(x, y)} \rangle$ is conservative if $f_x = g_y$.

iv. The circulation of a conservative vector field along a smooth, oriented curve is equal to zero.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 9. The part of the unit sphere with $x \leq 0$ and $z \leq 0$ is parameterized by $\mathbf{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$; the domain for this parameterization is equal to:

A. $D = \{ \ (\theta, \phi) \mid 0 \leq \theta \leq 2\pi, \ 0 \leq \phi \leq \pi \ \}$ B. $D = \{ \ (\theta, \phi) \mid \pi/2 \leq \theta \leq \pi, \ 0 \leq \phi \leq \pi/2 \ \}$

C. $D = \{ \ (\theta, \phi) \mid \pi \leq \theta \leq 2\pi, \ 0 \leq \phi \leq \pi/2 \}$ D. $D = \{ \ (\theta, \phi) \mid \pi/2 \leq \theta \leq 3\pi/2, \ \pi/2 \leq \phi \leq \pi \ \}$

E. $D = \{ \ (\theta, \phi) \mid \pi \leq \theta \leq 2\pi, \ \pi/2 \leq \phi \leq \pi \ \}$

(5 pts) 10. Let $f(x, y, z) = z + y^2$ and let S be the surface parameterized by $\mathbf{r}(\mathrm{u, v}) = \langle 2\mathrm{u}, -3\mathrm{v}, \mathrm{u} + \mathrm{v} \rangle$ with $0 \leq u \leq 1, \ 0 \leq v \leq 2$. The integral $\int \int_S \ f(x, y, z) \ dS$ is equal to:

A. 0 B. 160 C. 189 D. 84 E. 62

(5 pts) 11. Let S be the surface parameterized by $\mathbf{r}(\mathrm{u},\mathrm{v}) = \langle 2\mathrm{v}, \mathrm{u}^2+\mathrm{v}^2, \mathrm{u}\rangle$ with $0 \le u \le 2,\ 0 \le v \le 2$. A vector normal to the tangent plane of the surface at $u=1,\ v=1$ is:

A. $\langle -2, 2, -4\rangle$ B. $\langle 1, -2, 2\rangle$ C. $\langle 1, 2, -4\rangle$ D. $\langle -2, 1, -4\rangle$ E. $\langle 2, 2, -4\rangle$

(5 pts) 12. Let $f(x,y,z) = xye^z$ and let S be the part of the paraboloid $z = x^2+y^2+8$ where $x^2+y^2 \le 5$. The integral $\int\int_S f(x,y,z)\, dS$ is equal to:

A. $\int_{-5}^{5}\int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} xye^z\, dy\, dx$

B. $\int_{-\sqrt{5}}^{\sqrt{5}}\int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} xye^{x^2+y^2+8}\, dy\, dx$

C. $\int_{-5}^{5}\int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} xye^{x^2+y^2+8}\sqrt{4x^2+4y^2+1}\, dy\, dx$

D. $\int_{-\sqrt{5}}^{\sqrt{5}}\int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} xye^{x^2+y^2+8}\sqrt{2x+2y+1}\, dy\, dx$

E. none of the above

(5 pts) 13. Let $\mathbf{F} = \langle \mathrm{z}, \mathrm{x}, \mathrm{y}\rangle$ and let S be the surface parameterized by $\mathbf{r}(\mathrm{u},\mathrm{v}) = \langle \cos \mathrm{u}, \sin \mathrm{u}, 2\mathrm{v}+3\rangle$ with domain $R = \{\ (u,v) \mid 0 \le u \le 2\pi,\ 0 \le v \le 2\ \}$ and oriented so that normal vectors to the surface are pointing away from the z-axis; the integral $\int\int_S \mathbf{F}\cdot\hat{\mathbf{n}}\, \mathrm{dS}$ is given by:

A. $\int_0^{2\pi}\int_0^2 6v\cos u + 6\sin u\ dv\ du$

B. $\int_0^{2\pi}\int_0^2 6\sin u + 6v + v\sin u\cos u\ dv\ du$

C. $\int_0^{2\pi}\int_0^2 4v\cos u + 6\cos u + 2\sin u\cos u\ dv\ du$

D. $\int_0^{2\pi}\int_0^2 6\cos u + \sin u\cos u\ dv\ du$

E. $\int_0^{2\pi}\int_0^2 12v\cos u\sin u\ dv\ du$

(5 pts) 14. Let $\mathbf{F} = \langle \mathrm{x}, \mathrm{y}, \mathrm{z} \rangle$ and let S be the part of the plane $x + y + z = 1$ defined above the triangular region in the x, y-plane with vertices $(0,0)$, $(1,0)$, and $(0,1)$ and oriented so that the z component of the normal vector is positive; the flux of $\mathbf{F}$ across S is given by:

A. $\int_0^1 \int_0^1 1 \, dy \, dx$ B. $\int_0^1 \int_0^{-x+1} 1 \, dy \, dx$ C. $\int_0^1 \int_0^{-x+1} x + y \, dy \, dx$

D. $\int_0^1 \int_0^{-x+1} 1 - x - y \, dy \, dx$ E. none of the above

(5 pts) 15. Let C be the closed curve in the x, y-plane given by the triangle with vertices $(-1,0)$, $(0,2)$, and $(1,0)$ and oriented counter-clockwise. If $\mathbf{F} = \langle 4\mathrm{y} - 1, 3 - 2\mathrm{x} \rangle$, then the circulation of $\mathbf{F}$ around C is equal to:

A. 0 B. -6 C. 12 D. -12 E. 6

(5 pts) 16. Let $\mathbf{F} = \langle 2\mathrm{xy} + 4\mathrm{z}^2, \mathrm{x}^2 + 2\mathrm{z}, 2\mathrm{y} + 8\mathrm{xz} \rangle$; the divergence of $\mathbf{F}$ at the point $(x, y, z) = (1, 1, 1)$ is equal to:

A. 0 B. 4 C. 10 D. 12 E. 6

(5 pts) Bonus 17. The vector field in problem 16 is conservative.

A. True B. False

(5 pts) 18. Let $\mathbf{F} = \langle \mathrm{xz}, \mathrm{xy}, \mathrm{yz} \rangle$; the curl of $\mathbf{F}$ at the point $(x, y, z) = (0, 0, 0)$ is equal to:

A. $\langle -1, -1, -1 \rangle$ B. $\langle 1, 1, 1 \rangle$ C. $\langle 1, -1, 1 \rangle$ D. $\langle 0, 0, 0 \rangle$ E. $\langle 1, 1, 1 \rangle$

(5 pts) Bonus 19. The vector field in problem 18 is conservative.

A. True B. False

(5 pts) 20. Let S be a closed surface bounded by a cylinder of radius one centered on the z-axis, $z = 0$, and $z = 2$ and let $\mathbf{F} = \langle -6\mathrm{y}, 2\mathrm{x}, \mathrm{z}^2 \rangle$, then the flux across the surface is equal to:

A. 0 B. π C. 4π D. 3π E. 2π

(5 pts) 21. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ with $z \geq 0$ and oriented so that the z component of the normal vector is positive; if $\mathbf{F} = \left\langle -\mathrm{y} + \mathrm{z}, \mathrm{x}, \mathrm{e}^{\mathrm{x}^2 + \mathrm{y}^2} \right\rangle$, then the integral $\int \int_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} \, \mathrm{dS}$ is equal to:

A. 0 B. 6π C. 3π D. 2π E. 18π

(5 pts) 22. Which of the following are true:

I. The Divergence Theorem is given by $\int \int \int_D \mathbf{F} \cdot \hat{\mathbf{n}} \, \mathrm{dV} = \int \int_{\mathrm{S}} \nabla \cdot \mathbf{F} \, \mathrm{dS}$.

II. Stoke's Theorem is given by $\int_C \mathbf{F} \cdot \mathbf{T} \, \mathrm{ds} = \int \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} \, \mathrm{dS}$.

III. If $\mathbf{F}$ is conservative and S is a smooth bounded surface, then $\int \int_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} \, \mathrm{dS} = 0$.

IV. The flux of a constant vector field out of a solid is always greater than zero. (Hint: consider the Divergence Theorem)

A. only II and III B. only I and III C. only I, II, and IV D. only I, II, and III

E. I, II, III, and IV