Spring 2015

MAC2313 Final B

(5 pts) 1. Let ϕ be a twice differentiable potential function, $\mathbf{F}$ a twice differentiable vector field in R^3, and C a simple, smooth, closed, curve; how many of the following are true?

i. $\nabla \cdot \nabla\phi$ is equal to the scalar zero.

ii. $\oint_C \nabla\phi \cdot d\mathbf{r} = 0$.

iii. $\nabla \cdot (\nabla \times \mathbf{F})$ is equal to the zero vector.

iv. $\nabla \times (\nabla\phi)$ is equal to the zero vector.

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 2. Given a differentiable vector field $\mathbf{F}$ in R^3 and a smooth surface S; the Divergence Theorem is only applicable if the surface is closed.

A. true B. false

(5 pts) 3. Let D be the box formed by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, and $z = 3$ and let $\mathbf{F} = \langle 6\mathrm{y}, 9\mathrm{y}^2, \mathrm{z}^3\rangle$; the flux of $\mathbf{F}$ across the surface of the solid is equal to:

A. 0 B. 144 C. 162 D. 224 E. 306

(5 pts) 4. Let $\mathbf{F} = \langle \sin(2\mathrm{x}), 3\mathrm{yz}, 2\mathrm{x} + 3\mathrm{z}\rangle$, the divergence of $\mathbf{F}$ at the point $(\pi, 2, -1)$ is equal to:

A. 0 B. 2 C. 4 D. 6 E. none of the above

(5 pts) 5. Let $\mathbf{F} = \langle \sin(2x), 3yz, 2x + 3z \rangle$, the curl of $\mathbf{F}$ at the point $(\pi, 2, 1)$ is equal to:

A. $\langle 0, 0, 0 \rangle$ B. $\langle 6, -2, 0 \rangle$ C. $\langle -3, -2, 0 \rangle$ D. $\langle -3, 2, 0 \rangle$ E. $\langle -6, -2, 0 \rangle$

(5 pts) 6. Let S be a smooth oriented surface with a smooth boundary curve C ~~who's~~ whose orientation is consistent with the surface; if $\mathbf{F}$ is a differentiable vector field, then $\int\int_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}}\, dS = \oint_C \mathbf{F} \cdot \mathbf{T}\, ds$.

A. true B. false

(5 pts) 7. If $f(x, y, z) = (3x - 2y)z$ and C is the directed line segment with initial point $(1, 0, -1)$ and terminal point $(0, 2, 0)$, then $\int_C f(x, y, z)\, ds$ is equal to:

A. 0 B. $-2\sqrt{6}/3$ C. $-\sqrt{6}/3$ D. $-5\sqrt{6}/3$ E. none of the above

(5 pts) 8. If $\mathbf{F} = \langle 2x, 4y \rangle$ and C is the part of the parabola $y = x^2$ with initial point $(0, 0)$ and terminal point $(3, 9)$, then the vector line integral of $\mathbf{F}$ along C has a value of:

A. 0 B. 33 C. 66 D. 99 E. none of the above

(5 pts) 9. If $\mathbf{F} = \langle 4xy, 2y - 2 \rangle$ and C is given parametrically by $\mathbf{r}(t) = \langle 5\cos t, 3\sin t \rangle$, $t : 0 \to (\pi/2)$, then $\int_C \mathbf{F} \cdot d\mathbf{r}$ is equal to:

A. -200 B. -173 C. -100 D. -97 E. 0

(5 pts) 10. Let D be the annular region in the x, y-plane which lies between the circles $x^2+y^2=1$ and $x^2+y^2=4$; which of the following are true:

I. D is connected.

II. D is simply connected.

III. The curve $x^2+y^2=4$ is closed and simple.

IV. The curve $x^2+y^2=1$ is closed but not simple.

A. only III B. only IV C. only III and IV D. only I and III E. only I and IV

(5 pts) 11. Let $\mathbf{F} = \langle \mathrm{f(x,y), g(x,y)} \rangle$ with f and g differentiable and let R be a simply connected region in the x, y-plane with a smooth counterclockwise oriented boundary curve C, then $\int\int_R f_x - g_y \, dA = \int_C \mathbf{F} \cdot \mathbf{T} \, \mathrm{ds}$.

A. true B. false

(5 pts) 12. Let C be curve given by the triangle with vertices $(1,1)$, $(5,1)$ and $(5,3)$ with a counterclockwise orientation and let $\mathbf{F} = \langle \mathrm{y}+2, -3\mathrm{x}+7 \rangle$; the circulation of $\mathbf{F}$ around C is equal to:

A. 0 B. 32 C. 16 D. −32 E. −16

(5 pts) 13. Let $\mathbf{F} = \langle \mathrm{f(x,y,z), g(x,y,z), h(x,y,z)} \rangle$; $\mathbf{F}$ is called irrotational if $\nabla \cdot \mathbf{F} = 0$.

A. true B. false

(5 pts) 14. Let S be the upper half of a unit sphere oriented so that the z component of the unit normal vector is positive and let $\mathbf{F} = \langle \mathrm{y}, \mathrm{z}, \mathrm{x} \rangle$. What is the value of $\int\int_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}}\, \mathrm{dS}$? (Hint: do not calculate this directly.)

A. 0 B. -2π C. $-\pi$ D. 2π E. π

(5 pts) 15. The vector field $\mathbf{F} = \langle 2\mathrm{xy} - 3\mathrm{yz}^2, \mathrm{x}^2 - 3\mathrm{z}^2, 6\mathrm{xyz} \rangle$ is conservative.

A. true B. false

(5 pts) 16. How many of the following vector fields are incompressible?

i. $\mathbf{F} = \langle \mathrm{x}, \mathrm{y}, \mathrm{z} \rangle$

ii. $\mathbf{F} = \langle \mathrm{x}^2, 0, \mathrm{xe}^{\mathrm{y}} \rangle$

iii. $\mathbf{F} = \langle \mathrm{z}, \mathrm{x}, \mathrm{y} \rangle$

iv. $\mathbf{F} = \left\langle 4\mathrm{y}\cos^2\mathrm{x}, -2\mathrm{y}^2 \sin\mathrm{x}\cos\mathrm{x}, \sqrt{\mathrm{x}^2 + \mathrm{y}^2} \right\rangle$

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 17. Let S be the part of the parabaloid $z = 1 - x^2 - y^2$ with $z \geq 0$ and oriented so that the z component of the normal vector is positive. If $\mathbf{F} = \langle \mathrm{x}, \mathrm{y}, 1 \rangle$, the flux of $\mathbf{F}$ across S is equal to:

A. 0 B. 1 C. $\pi/2$ D. π E. 2π

(5 pts) 18. If $f(x, y, z) = 4$ and S is the part of the plane $3x + 3y + z = 9$ which lies in the first octant, then $\int\int_S f(x, y, z)\, dS$ is equal to:

A. 0 B. $3\sqrt{19}$ C. $9\sqrt{19}$ D. $18\sqrt{19}$ E. $27\sqrt{19}$

(5 pts) 19. Let the surface S be the graph of the function $z = 2x^2 - 3y$, a normal vector to S at the point $(1, -1, 5)$ is given by:

A. $\langle -4, 3, 1\rangle$ B. $\langle -4, -3, 1\rangle$ C. $\langle 4, -3, 1\rangle$ D. $\langle 4, 3, 1\rangle$ E. none of the above

(5 pts) 20. Lets S be the part of a cylinder parametrized by $\mathbf{r}(\theta, \mathrm{z}) = \langle 3\cos\theta, 3\sin\theta, \mathrm{z}\rangle$, $R = \{\ (\theta, z)\ |\ 0 \leq \theta \leq 2\pi,\ 0 \leq z \leq 1\ \}$ and oriented so that the normal vector points away from the z axis. If $\mathrm{F} = \langle \mathrm{xz}, \mathrm{yz}, \mathrm{x} + \mathrm{y}\rangle$, then the integral $\int\int_S \mathrm{F} \cdot \hat{\mathrm{n}}\ \mathrm{dS}$ is equal to:

A. 0 B. 9π C. 18π D. 24π E. 36π

(5 pts) Bonus 21. Lets S be the part of a cylinder parametrized by $\mathbf{r}(\theta, z) = \langle 3\cos\theta, 3\sin\theta, z\rangle$, $R = \{\ (\theta, z) \mid 0 \le \theta \le 2\pi,\ 0 \le z \le 2\ \}$. The integral $\int\int_S x^2 + y\ dS$ is equal to:

A. $\int_0^{2\pi}\int_0^2 (27\cos^2\theta + 9\sin\theta)\ dz\ d\theta$ B. $\int_0^{2\pi}\int_0^2 \sqrt{3}(9\cos^2\theta + 3\sin\theta)\ dz\ d\theta$

C. $\int_0^{2\pi}\int_0^2 (9\cos^2\theta{+}3\sin\theta)(3\sin\theta\cos\theta)\ dz\ d\theta$ D. $\int_0^{2\pi}\int_0^2 (9\cos^2\theta{+}3\sin\theta)\sqrt{3\cos\theta + 3\sin\theta}\ dz\ d\theta$

E. none of the above

(5 pts) Bonus 22. Let D be the tetrahedron formed by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ and let $\mathbf{F} = 18z\mathbf{k}$; the flux of $\mathbf{F}$ across the surface of the solid is equal to:

A. 0 B. 2 C. 3 D. 6 E. 12