

Practice Problems - Lecture 17

Problem 1. Sketch the graph of the rational function (including vertical and horizontal asymptotes), and state the domain, range, and intervals on which the function is increasing and decreasing:

(a) $f(x) = \frac{1}{x-3}$;

(b) $f(x) = \frac{-1}{x} - 2$;

(c) $f(x) = \frac{1}{(x+3)^2}$;

(d) $f(x) = \frac{-1}{(x-4)^2} + 2$.

Problem 2. Find the equations of any vertical or horizontal asymptotes, the coordinates of any holes, and the zeros of each rational function:

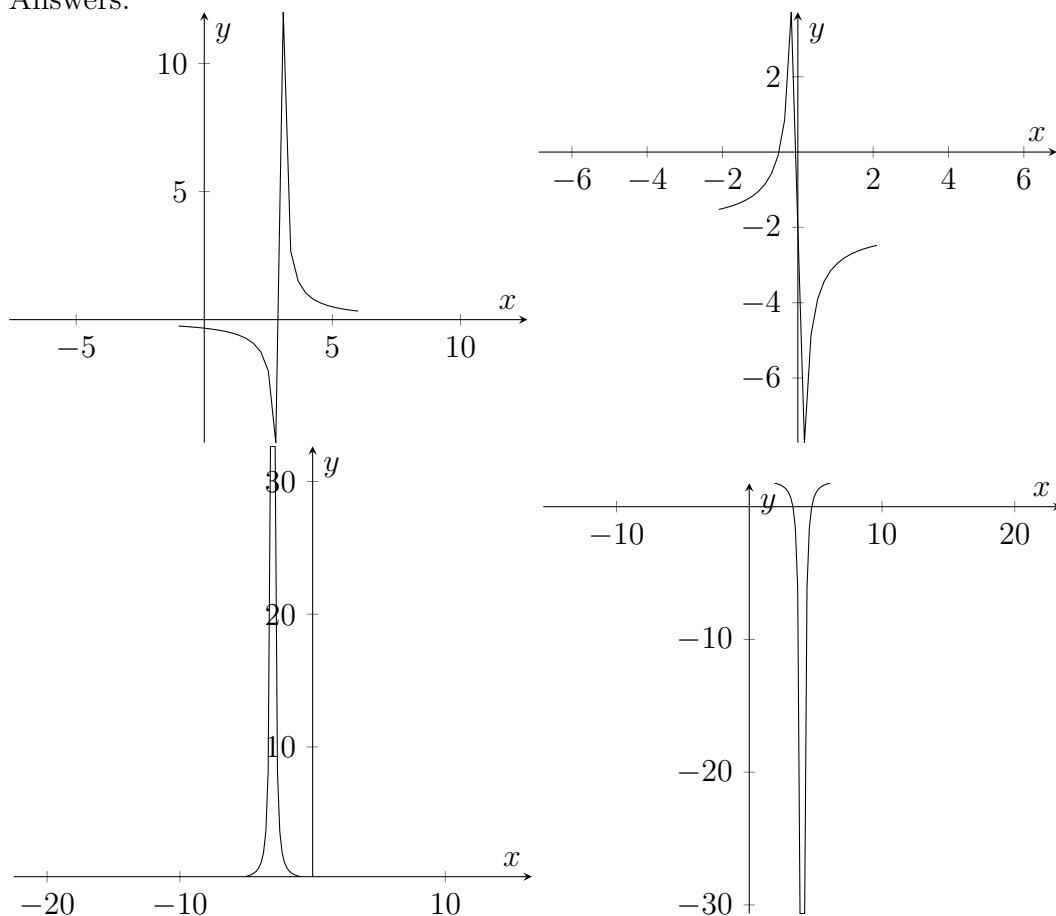
(a) $f(x) = \frac{-6}{x+9}$;

(b) $f(x) = \frac{2x+6}{x-4}$;

(c) $f(x) = \frac{x^2-1}{x-1}$;

(d) $f(x) = \frac{x^2-2x-3}{2x^2-3x-5}$.

Answers:



1. (a) Domain: $x \neq 3$, Range: $y \neq 0$, Increasing: $\emptyset$, Decreasing: $(-\infty, 3) \cup (3, \infty)$;
- (b) Domain: $x \neq 0$, Range: $y \neq -2$, Increasing: $(-\infty, 0) \cup (0, \infty)$, Decreasing: $\emptyset$;
- (c) Domain: $x \neq -3$, Range: $(0, \infty)$, Increasing: $(-\infty, -3)$, Decreasing: $(-3, \infty)$;
- (d) Domain: $x \neq 4$, Range: $(-\infty, 2)$, Increasing: $(4, \infty)$, Decreasing: $(-\infty, 4)$.

2. (a) VA: $x = -9$, HA: $y = 0$, no holes, no zeros;
- (b) VA: $x = 4$, HA: $y = 2$, no holes, zero at $x = -3$;
- (c) VA: none, HA: none, hole at $(1, 2)$, zero at $x = -1$;
- (d) VA: $x = \frac{5}{2}$, HA: $y = \frac{1}{2}$, hole at $\left(-1, \frac{4}{7}\right)$, zero at $x = 3$.