Practice Problems - Lecture 19

Instructions: On a separate sheet of paper, complete the following problems by writing out full solutions. To receive full credit, you must show enough work to demonstrate mastery of the concepts. You may consult your notes, work with other students, and ask me for help during class or office hours, but your submitted work should be yours alone according to the UF honor code. This counts as participation grade 6, is due at the beginning of class on Tuesday, November 17 and is worth 5 points.

Problem 1. Let $f(x) = x^2 + 3$ and $g(x) = -2x + 6$. Evaluate the following:
(a) $(f + g)(-5)$; (b) $(f - g)(4)$; (c) $(fg)(-3)$; (d) $\left(\frac{f}{g}\right)(3)$.

Problem 2. For each pair of functions, find the given functions and their domains:
(a) $f(x) = 6 - 3x, g(x) = -4x + 1$; find $(f + g)(x)$;
(b) $f(x) = 4x^2 + 2x, g(x) = x^2 - 3x + 2$; find $\left(\frac{f}{g}\right)(x)$;
(c) $f(x) = \sqrt{5x + 4}, g(x) = -\frac{1}{x}$; find $(fg)(x)$.

Problem 3. Use the given table to evaluate the following expressions:

x	$f(x)$	$g(x)$
-2	-4	2
0	8	-1
2	5	4
4	0	0

(a) $(f + g)(2)$; (b) $(f - g)(4)$; (c)$(fg)(-2)$; (d) $\left(\frac{f}{g}\right)(0)$.

Problem 4. Let $f(x) = 2x - 3$ and $g(x) = -x + 3$. Find the following values:
(a) $(f \circ g)(2)$; (b) $(g \circ f)(3)$; (c) $(g \circ g)(-2)$.

Problem 5. For each pair of functions, find both $(f \circ g)(x)$ and $(g \circ f)(x)$ as well as their respective domains:
(a) $f(x) = 8x + 12, g(x) = 3x - 1$;
(b) $f(x) = \sqrt{x}, g(x) = x - 1$;
(c) $f(x) = \sqrt{x - 2}, g(x) = 2x$;
(d) $f(x) = \sqrt{x}, g(x) = \frac{3}{x + 6}$.

Problem 6. Find functions f and g such that $h(x) = (f \circ g)(x)$:
(a) $h(x) = (11x^2 + 12x)^2$;
(b) $h(x) = (2x - 3)^3$;
(c) $h(x) = \sqrt[3]{2x + 3} - 4$.

Problem 7. The perimeter of a square with side length s is given by the formula $P = 4s$.
(a) Solve for s in terms of P.
(b) If A represents the area of this square, write A as a function of the perimeter P.
(c) Use the composite function $(A \circ s)(P)$ from part (b) to find the area of a square with perimeter 6.

Problem 8. At 8 A.M., a factory smokestack begins emitting a pollutant which disperses horizontally over a circular area. If t represents the time in hours since the factory began emitting pollutants ($t = 0$ represents 8 A.M.), assume that the radius of the circle of pollutants at time t is $r(t) = 2t$ miles. Let $A(r) = \pi r^2$ represent the area of a circle of radius r.
(a) Find $(A \circ r)(t)$.
(b) Interpret $(A \circ r)(t)$.
(c) What is the area of the circular region covered by the layer at noon?

Answers:

1. (a) 44; (b) 21; (c) 144; (d) undefined.

2. (a) $(f+g)(x) = -7x+7$ has domain $(-\infty, \infty)$;
(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{4x^2+2x}{x^2-3x+2}$ has domain $(-\infty,1)\cup(1,2)\cup(2,\infty)$;
(c) $(fg)(x) = \dfrac{-\sqrt{5x+4}}{x}$ has domain $\left[-\dfrac{4}{5}, 0\right)\cup(0,\infty)$.

3. (a) 9; (b) 0; (c) -8; (d) -8.

4. (a) -1; (b) 0; (c) -2.

5. (a) $(f\circ g)(x) = 24x+4$ has domain $(-\infty,\infty)$, $(g\circ f)(x) = 24x+35$ has domain $(-\infty,\infty)$;
(b) $(f\circ g)(x) = \sqrt{x-1}$ has domain $[1,\infty)$, $(g\circ f)(x) = \sqrt{x}-1$ has domain $[0,\infty)$;
(c) $(f\circ g)(x) = \sqrt{2x-2}$ has domain $[1,\infty)$, $(g\circ f)(x) = 2\sqrt{x-2}$ has domain $[2,\infty)$;
(d) $(f\circ g)(x) = \sqrt{\dfrac{3}{x+6}}$ has domain $(-6,\infty)$, $(g\circ f)(x) = \dfrac{3}{\sqrt{x}+6}$ has domain $[0,\infty)$.

6. (Other answers are possible) (a) $f(x) = x^2, g(x) = 11x^2+12x$;
(b) $f(x) = x^3$, $g(x) = 2x-3$;
(c) $f(x) = \sqrt[3]{x}-4$, $g(x) = 2x+3$.

7. (a) $s = \dfrac{P}{4}$; (b) $A(P) = \dfrac{P^2}{16}$; (c) $A(6) = \dfrac{9}{4}$.

8. (a) $(A\circ r)(t) = 4\pi t^2$; (b) $(A\circ r)(t)$ represents the area of the circular region covered by the pollutant after t hours; (c) 64π square miles.