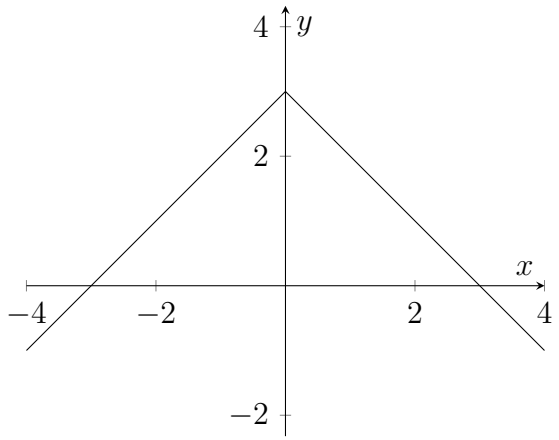
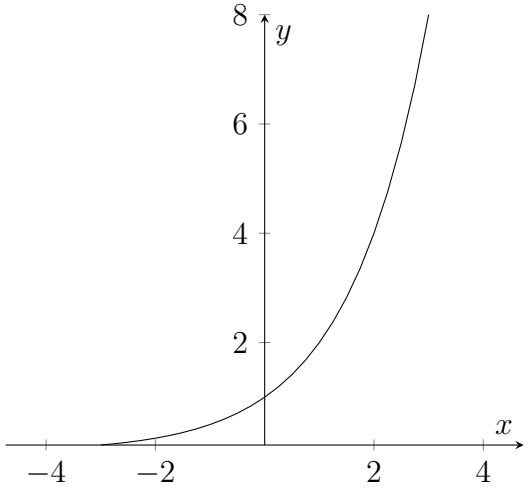
Practice Problems - Lecture 20

Problem 1. Determine whether the following functions are one-to-one:
(a) $y = 4x + 20$; (b) $y = -\sqrt{100 - x^2}$; (c) $y = \dfrac{4}{x - 8}$;
(d) $y = -3(x - 6)^2 + 8$ (hint: consider the graph);





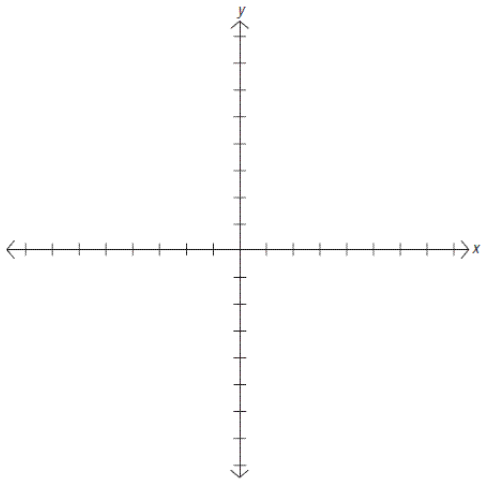
Problem 2. Use the definition of inverses to determine whether f and g are inverses:
(a) $f(x) = 3x + 9, g(x) = \dfrac{1}{3}x - 3$;
(b) $f(x) = -4x + 2, g(x) = -\dfrac{1}{4}x - 2$;
(c) $f(x) = \dfrac{x - 3}{x + 4}, g(x) = \dfrac{4x + 3}{1 - x}$.

Problem 3. Find the inverse of the function, if possible:
(a) $\left\{(3, -1), (5, 0), (0, 5), \left(4, \dfrac{2}{3}\right)\right\}$; (b) $\{(6, -8), (3, -4), (0, -8), (5, -4)\}$.

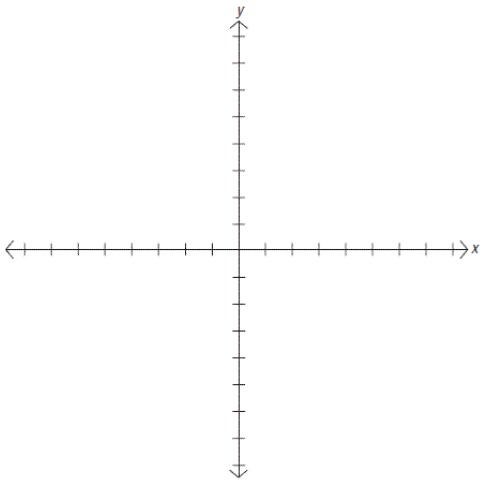
Problem 4. The following functions are all one-to-one. Write an equation for the inverse function $y = f^{-1}(x)$:
(a) $f(x) = 4x - 5$;
(b) $f(x) = -x^3 - 2$;
(c) $f(x) = \dfrac{1}{x + 2}, x \neq -2$;
(d) $f(x) = \dfrac{x + 2}{x - 2}, x \neq 2$.

Problem 5. Graph the inverse of each function on the same axes:

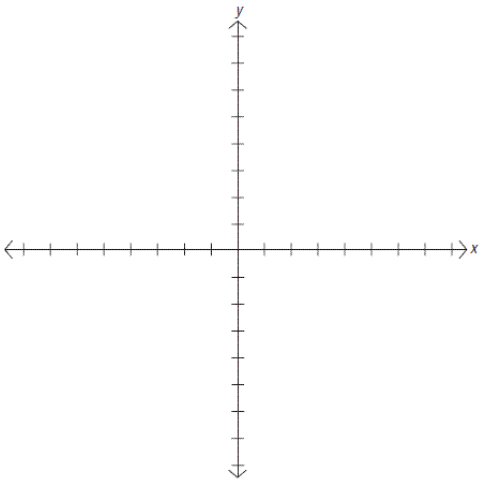
(a)



(b)



(c)



Problem 6. The standard cipher associates each letter of the alphabet to its number in alphabetical order: 1 goes with A, 2 goes with B, etc. (we assign 0 to the empty space). A message is encoded by converting the letters into numbers using the standard cipher, and then plugging each number of the message into the function $f(x) = 3x - 2$. The output is as follows:

$$37 \quad 1 \quad 58 \quad 22 \quad -2 \quad 25 \quad 55 \quad -2 \quad 16 \quad 61 \quad 40$$

To decode the message, find the inverse function of f, substitute each output number into this inverse, and translate back to letters using the standard cipher. What is the message?

Answers:

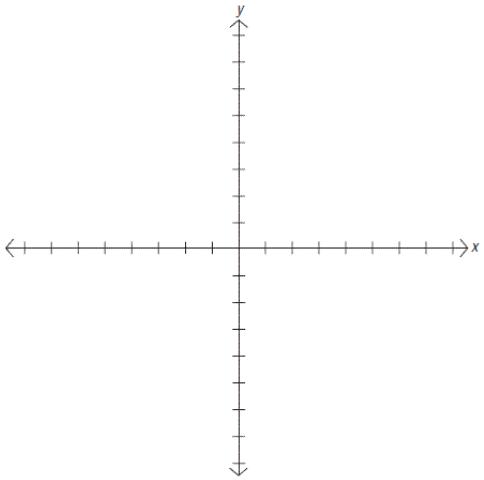
1. (a) Yes; (b) No; (c) Yes; (d) No; (e) No; (f) Yes.

2. (a) Yes; (b) No; (c) Yes.

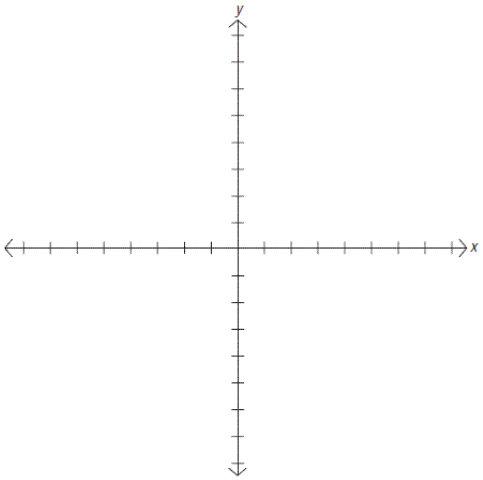
3. (a) $\left\{(-1,3),(0,5),(5,0),\left(\frac{2}{3},4\right)\right\}$;
(b) No inverse (y-values -8 and -4 are both repeated).

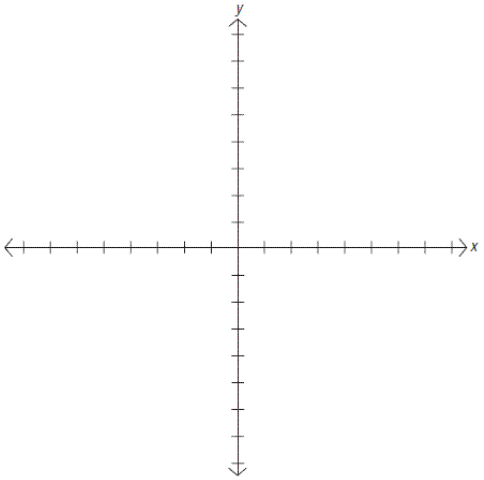
4. (a) $f^{-1}(x)=\frac{1}{4}x+\frac{5}{4}$;
(b) $f^{-1}(x)=\sqrt[3]{-x-2}$;
(c) $f^{-1}(x)=\frac{1}{x}-2, x\neq 0$;
(d) $f^{-1}(x)=\frac{2x+2}{x-1}, x\neq 1$.

5. (a)



(b)





(c)

6. MATH IS FUN