Practice Problems - Lecture 3

Problem 1. Find the following quotients using long division:
(a) $\dfrac{4m^3 - 8m^2 + 5m + 6}{2m - 1}$; (b) $\dfrac{3x^3 - 2x^2 - 150}{x^2 - 4}$.

Problem 2. Use synthetic division to find the quotient and remainder:
(a) $\dfrac{5x^3 - 6x^2 - 28x - 2}{x + 2}$;
(b) $\dfrac{x^3 - 4x^2 + 9x - 6}{x - 1}$;
(c) $\dfrac{x^4 + x^2 - 3x + 1}{x + 1}$.

Problem 3. Reduce the rational expression to lowest terms, and find the domain:
(a) $\dfrac{2x^2 + 7x - 4}{5x^2 + 20x}$; (b) $\dfrac{6 - 3x}{x^2 - 4}$.

Problem 4. Find the product or quotient as indicated:
(a) $\dfrac{2y^2}{9} \cdot \dfrac{27}{8y^5}$;
(b) $\dfrac{6r - 18}{3r^2 + 2r - 8} \cdot \dfrac{12r - 16}{4r - 12}$;
(c) $\dfrac{x^2 + 2x - 15}{x^2 + 11x + 30} \div \dfrac{x^2 - 8x + 15}{x^2 + 2x - 24}$.

Problem 5. Find the sum or difference:
(a) $\dfrac{8}{3p} + \dfrac{5}{4p} + \dfrac{9}{2p}$;
(b) $\dfrac{7}{9a^2b} - \dfrac{2}{18ab^2}$;
(c) $\dfrac{m + 1}{m - 1} + \dfrac{m - 1}{m + 1}$.

Problem 6. Simplify the complex fraction:
(a) $\dfrac{1 + \dfrac{1}{1 - b}}{1 - \dfrac{1}{1 + b}}$;
(b) $\dfrac{\dfrac{y + 3}{y} - \dfrac{4}{y - 1}}{\dfrac{y}{y - 1} + \dfrac{1}{y}}$.

Answers:

1. (a) $2m^2 - 3m + 1 + \dfrac{7}{2m-1}$; (b) $3x - 2 + \dfrac{12x - 158}{x^2 - 4}$.
2. (a) $5x^2 - 16x + 4 - \dfrac{10}{x+2}$; (b) $x^2 - 3x + 6$; (c) $x^3 - x^2 + 2x - 5 + \dfrac{6}{x+1}$.
3. (a) $\dfrac{2x-1}{5x}$, domain: $\{x \,|\, x \neq 0, x \neq -4\}$; (b) $\dfrac{-3}{x+2}$, domain: $\{x \,|\, x \neq 2, x \neq -2\}$.
4. (a) $\dfrac{3}{4y^3}$; (b) $\dfrac{6}{r+2}$; (c) $\dfrac{x-4}{x-5}$.
5. (a) $\dfrac{101}{12p}$; (b) $\dfrac{7b-a}{9a^2b^2}$; (c) $\dfrac{2(m^2+1)}{m^2-1}$.
6. (a) $\dfrac{(2-b)(1+b)}{b(1-b)}$; (b) $\dfrac{y^2 - 2y - 3}{y^2 + y - 1}$.