

Practice Problems - Lecture 7

Problem 1. Identify each number as real, pure imaginary, or nonreal complex:

- (a) $-7i$; (b) $-6 - 2i$; (c) $\sqrt{24}$; (d) $\sqrt{-36}$.

Problem 2. Multiply or divide:

- (a) $\sqrt{-17} \cdot \sqrt{-17}$;
(b) $\sqrt{-5} \cdot \sqrt{-15}$;
(c) $\frac{\sqrt{-70}}{\sqrt{-7}}$;
(d) $\frac{20 + \sqrt{-8}}{2}$.

Problem 3. Find the sum, difference, or product:

- (a) $(4 - i) + (8 + 5i)$;
(b) $(-2 + 4i) - (-4 + 2i)$;
(c) $(1 + 3i)(2 - 5i)$;
(d) $(2 + i)^2$;
(e) $(2 + 7i)(2 - 7i)$.

Problem 4. Find the quotient and write in standard form:

- (a) $\frac{6 + 2i}{1 + 2i}$;
(b) $\frac{2}{3i}$.

Problem 5. Simplify the powers of i :

- (a) i^{54} ; (b) i^{23} ; (c) i^{-13} ; (d) i^{40} .

Problem 6. Solve each quadratic equation:

- (a) $x^2 - 2x + 10 = 0$;
(b) $2x^2 - 4x + 3 = 0$.

Problem 7. Evaluate the discriminant, and use it to predict the number of distinct solutions, and whether they are rational, irrational, or nonreal complex:

- (a) $x^2 + 4x + 4 = 0$;
(b) $8x^2 = -14x - 3$;
(c) $2x^2 + 4x + 1 = 0$;
(d) $3x^2 = 4x - 5$.

Answers:

1. (a) pure imaginary; (b) nonreal complex; (c) real; (d) pure imaginary.

2. (a) -17 ; (b) $-5\sqrt{3}$; (c) $\sqrt{10}$; (d) $10 + i\sqrt{2}$.

3. (a) $12 + 4i$; (b) $2 + 2i$; (c) $17 + i$; (d) $3 + 4i$; (e) 53 .

4. (a) $2 - 2i$; (b) $-\frac{2}{3}i$.

5. (a) -1 ; (b) $-i$; (c) $-i$; (d) 1 .

6. (a) $1 \pm 3i$; (b) $1 \pm \frac{\sqrt{2}}{2}i$.

7. (a) $D = 0$, one solution, rational; (b) $D = 100$, two solutions, rational; (c) $D = 8$, two solutions, irrational; (d) $D = -164$, two solutions, nonreal complex.