MAC2313 Test 1 A Fall 2015

(5 pts) 1. Let $\mathbf{v}$ be a vector with initial point $(-1, -2, 3)$ and terminal point $(0, 4, -2)$. How many of the following are true?

$\vec{v} = \langle 1, 6, -5 \rangle$

i. $|\mathbf{v}| = \sqrt{62}$ ✓ $\|\vec{v}\| = \sqrt{1+36+25} = \sqrt{62}$

ii. The vector $\mathbf{v}$ is parallel to $\langle 2, -12, 10 \rangle$. × $2\vec{v} = \langle 2, 12, -10 \rangle$

iii. The vector $\mathbf{v}$ is orthogonal to $\langle 2, 2, 2 \rangle$. × $\langle 1, 6, -5 \rangle \cdot \langle 2, 2, 2 \rangle = 2 + 12 - 10 = 4 \neq 0$

iv. $\mathbf{v} = -\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$ × $\hat{i} + 6\hat{j} - 5\hat{k}$

A. 0 $\boxed{\text{B. 1}}$ C. 2 D. 3 E. 4

(5 pts) 2. The point $(-14, -12, 11)$ lies on the line $\mathbf{r}(t) = \langle 2 - 4t, -3t, 2t + 3 \rangle$, $t \in (-\infty, \infty)$.

$2 - 4t = -14$
$-4t = -16$
$t = 4$?

$\vec{r}(4) = \langle -14, -12, 11 \rangle$

$\boxed{\text{A. True}}$ B. False

(5 pts) 3. Which of the following is an equation of the line segment connecting the points $(2, 1, -2)$ and $(3, -1, 4)$?

$\vec{r}(t) = (1-t)\langle 2, 1, -2 \rangle + t\langle 3, -1, 4 \rangle$
$= \langle 2 + t, 1 - 2t, -2 + 6t \rangle$

A. $\mathbf{r}(t) = \langle 2 + t, 1 - t, 2 - 6t \rangle$, $t \in [0, 1]$

B. $\mathbf{r}(t) = \langle 2 + t, 2t, 2 + 6t \rangle$, $t \in [0, 1]$

C. $\mathbf{r}(t) = \langle 2 - t, -1 + 2t, -2 - 6t \rangle$, $t \in [0, 1]$

$\boxed{\text{D.}}$ $\mathbf{r}(t) = \langle 2 + t, 1 - 2t, -2 + 6t \rangle$, $t \in [0, 1]$

E. none of the above

(5 pts) 4. What is the value of $[\langle 2,0,1\rangle \times \langle 0,-1,3\rangle] \cdot \langle 1,-1,2\rangle$?

$\boxed{\text{A. } 3}$ B. -8 C. 1 D. -1 E. 6

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 0 & -1 & 3 \end{vmatrix} = \langle 1,-6,-2\rangle \cdot \langle 1,-1,2\rangle = 1+6-4 = 3$

(5 pts) 5. The point $(0,1,-1)$ lies on the line of intersection of the planes $x+y+z=0$ and $-x+y+z=0$; an equation of the line of intersection is given by:

$\vec{n}_1 = \langle 1,1,1\rangle, \quad \vec{n}_2 = \langle -1,1,1\rangle$

A. $\mathbf{r}(t) = \langle t, 1-2t, 1+2t\rangle$, $t \in (-\infty, \infty)$

$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \langle 0,-2,2\rangle$

B. $\mathbf{r}(t) = \langle 0, 1-2t, 1+2t\rangle$, $t \in (-\infty, \infty)$

$\vec{r}(t) = \langle 0,1,-1\rangle + t\langle 0,-2,2\rangle$

$= \langle 0, 1-2t, -1+2t\rangle$

C. $\mathbf{r}(t) = \langle 1, 1-2t, 1+2t\rangle$, $t \in (-\infty, \infty)$

$\boxed{\text{D.}}$ $\mathbf{r}(t) = \langle 0, 1-2t, -1+2t\rangle$, $t \in (-\infty, \infty)$

E. none of the above

(5 pts) 6. If $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ are nonzero vectors in R^3 and λ is a scalar, then which of the following are true?

I. $\mathbf{u} \cdot \mathbf{u} > 0$ ✓

$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2 > 0$ since $\vec{u}$ is nonzero

II. If $\mathbf{v}$ and $\mathbf{w}$ are parallel then $\mathbf{v} \times \mathbf{w} = \mathbf{0}$. ✓

$\vec{v} \times \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \sin\theta = 0$ since $\theta = 0$.

III. $(\mathbf{u}+\mathbf{v}) \times \mathbf{w} = (\mathbf{w} \times \mathbf{u}) + (\mathbf{w} \times \mathbf{v})$ ✗

$= (\vec{u} \times \vec{w}) + (\vec{v} \times \vec{w})$

IV. $(\lambda\mathbf{u}) \times \mathbf{v} = -[\mathbf{u} \times (\lambda\mathbf{v})]$ ✗

$= \vec{u} \times (\lambda\vec{v})$

A. only II B. only I and III $\boxed{\text{C.}}$ only I and II D. only I, II, and III E. only I, III, and IV

(5 pts) 7. The domain of the function $\mathbf{r}(t) = \left\langle \sqrt{t^2-1}, \ln t^2, 1/(1-t) \right\rangle$ is given by:

$t^2 - 1 \ge 0 \Rightarrow t^2 \ge 1 \Rightarrow |t| \ge 1$ $\quad t^2 > 0 \Rightarrow t \ne 0 \quad t \ne 1$

A. $(-\infty, -1] \cup (1, +\infty)$ B. $[-1, 0) \cup (0, 1)$ C. $(0, 1) \cup (1, +\infty)$ D. $[1, +\infty)$ E. $(1, +\infty)$

$\Rightarrow |t| > 1$ or $(-\infty, -1] \cup (1, \infty)$

(5 pts) 8. If $\mathbf{r}(t) = \langle -3t^2, 8, -t^3 \rangle$, then the length of the curve on the interval $[0, 1]$ is

$\vec{r}\,'(t) = \langle -6t, 0, -3t^2 \rangle \Rightarrow \|\vec{r}\,'(t)\| = \sqrt{36t^2 + 9t^4} = t\sqrt{36 + 9t^2}$

A. $(\sqrt{5})^3 - 4$ B. $(\sqrt{5})^3 - 2$ C. $\sqrt{5} - 8$ D. $\boxed{(\sqrt{5})^3 - 8}$ E. $\sqrt{5} - 4$

$\frac{1}{18}\int_0^1 18t\sqrt{36+9t^2}\,dt = \frac{1}{18}\int_{36}^{45}\sqrt{u}\,du = \frac{2}{3}\cdot\frac{1}{18}u^{3/2}\Big|_{36}^{45} = \frac{1}{27}\left(\sqrt{45}^3 - \sqrt{36}^3\right) = \frac{3^3\sqrt{5}^3 - 6^3}{3^3}$

$u = 9t^2 + 36 \quad du = 18t\,dt$

$= (\sqrt{5})^3 - 8$

(5 pts) 9. If $\mathbf{r}(t) = \langle e^{2t}, 6\cos(3t), 1/(2t+1) \rangle$, then $\int_0^{\pi/2} \mathbf{r}(t)\,dt$ is equal to:

A. $\langle (1/2)(e^{\pi} - 1), 0, \ln(\pi + 1) \rangle$

B. $\langle (1/2)(e^{2\pi} - e), 0, \ln(\pi + 1) \rangle$

C. $\boxed{\langle (1/2)(e^{\pi} - 1), -2, (1/2)\ln(\pi + 1) \rangle}$

D. $\langle e^{2\pi} - e, -2, \ln(\pi + 1) - 1 \rangle$

E. $\langle (1/2)(e^{2\pi} - 1), 2, (\ln(\pi + 1) \rangle$

$\int_0^{\pi/2} \langle e^{2t}, 6\cos(3t), \frac{1}{2t+1} \rangle dt = \langle \frac{1}{2}e^{2t}, 2\sin(3t), \frac{1}{2}\ln(2t+1) \rangle \Big|_0^{\pi/2} = \langle \frac{1}{2}e^{\pi}, -2, \frac{1}{2}\ln(\pi+1) \rangle - \langle \frac{1}{2}, 0, 0 \rangle = \langle \frac{1}{2}e^{\pi} - \frac{1}{2}, -2, \frac{1}{2}\ln(\pi+1) \rangle$

(5 pts) 10. Which of the following vectors is orthogonal to the normal vector of the plane $2x - 3y + z = 5$?

$\vec{n} = \langle 2, -3, 1 \rangle$

A. $\langle 1, 0, 2 \rangle$ B. $\langle 4, -6, 1 \rangle$ C. $\langle 4, 6, -1 \rangle$ D. $\langle -4, -6, -1 \rangle$ E. $\boxed{\langle 1, 1, 1 \rangle}$

4 $\quad 8 + 18 + 1 \quad 8 - 18 - 1 \quad -8 + 18 - 1$

(5 pts) 11. How many of the following are true?

i. The curvature of a straight line is zero. ✓

ii. $\kappa = \dfrac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3}$ ✓

iii. $\mathbf{B}\cdot\mathbf{N} = 0$ ✓

iv. $|\mathbf{r}'(\mathrm{t})| = \dfrac{ds}{dt}$ ✓

A. 0 B. 1 C. 2 D. 3 $\boxed{\text{E. 4}}$

(5 pts) 12. A box is placed on a flat surface and constant force $\mathbf{F}$ is applied to the box at an angle θ with respect to the horizontal; if the displacement of the box is given by the vector $\mathbf{d}$, then the work done in moving the box is equal to $|\mathbf{F}||\mathbf{d}|\sin\theta$.

$W = \vec{F}\cdot\vec{d} = \|\vec{F}\|\cdot\|\vec{d}\|\cos\theta$

A. True $\boxed{\text{B. False}}$

(5 pts) 13. Let $\mathbf{u} = \langle 4, 6, -2\rangle$ and $\mathbf{v} = \langle 1, 2, -1\rangle$; the projection of $\mathbf{u}$ onto $\mathbf{v}$ is given by:

A. $\langle 0,0,0\rangle$ B. $\langle 1,2,-1\rangle$ C. $\langle 2,4,-2\rangle$ D. $\langle -3,6,-3\rangle$ $\boxed{\text{E. none of the above}}$

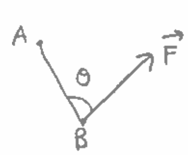
$\vec{u}_{\parallel} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\frac{4+12+2}{1+4+1}\right)\vec{v} = \left(\frac{18}{6}\right)\vec{v} = 3\vec{v} = \langle 3, 6, -3\rangle$

(5 pts) 14. Bonus. Let A and B be separate points in R^3 and let $\mathbf{F}$ be a constant force applied at the point B; the magnitude of the resulting torque about the point A is least when the angle (in radians) between $\mathbf{F}$ and $\vec{\mathbf{AB}}$ is (is minimum)

A. 0 B. $\pi/6$ C. $\pi/4$ D. $\pi/3$ E. $\pi/2$

$\|\vec{\tau}\| = \|\vec{F}\|\|\vec{AB}\|\sin\theta = 0$

when $\sin\theta = 0 \Rightarrow \theta = 0$.



MAC2313 Test 1A

Name: ______________________ UF-ID: ________________ Section: ______________

(7 pts) 1. If $\mathbf{r}(t) = \langle 2, -3t, 2t^2 \rangle$, what is the curvature when $t = 1$?

$\vec{v}(t) = \langle 0, -3, 4t \rangle$

$\vec{a}(t) = \langle 0, 0, 4 \rangle$

$\Rightarrow \quad \vec{v} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & 4t \\ 0 & 0 & 4 \end{vmatrix} = \langle -12, 0, 0 \rangle$

$\Rightarrow \|\vec{v} \times \vec{a}\| = 12$

$\|\vec{v}(t)\| = \sqrt{9+16t^2} \quad \Rightarrow \quad \kappa = \dfrac{\|\vec{v} \times \vec{a}\|}{\|\vec{v}(t)\|^3} = \dfrac{12}{(16t^2+9)^{3/2}}$

$K(1) = \dfrac{12}{(16+9)^{3/2}} = \dfrac{12}{(\sqrt{25})^3} = \boxed{\dfrac{12}{125}}$

(7 pts) 2. Let $\mathbf{u} = \langle 1, 0, 1 \rangle$ and $\mathbf{v} = \langle 1, 2, 1 \rangle$; if we decompose $\mathbf{u}$ as $\mathbf{u} = \mathbf{u}_{\|} + \mathbf{u}_{\perp}$, what is the value of $\|\mathbf{u}_{\perp}\|$?

$\vec{u}_{\|} = \left(\dfrac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\right)\vec{v} = \left(\dfrac{1+1}{1+4+1}\right)\vec{v} = \dfrac{1}{3}\langle 1, 2, 1 \rangle = \langle \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{3} \rangle$

$\vec{u}_{\perp} = \langle 1, 0, 1 \rangle - \langle \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{3} \rangle = \langle \tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{2}{3} \rangle$

$\|\vec{u}_{\perp}\| = \dfrac{2}{3}\|\langle 1, -1, 1 \rangle\| = \dfrac{2}{3}\sqrt{3} = \boxed{\dfrac{2}{\sqrt{3}}}$

$\sqrt{\tfrac{12}{9}} = \tfrac{2}{3}\sqrt{\tfrac{3}{1}}$

(7 pts) 3. What is the area of the parallelogram generated by the vectors $\mathbf{u} = \langle 0, 3, 1 \rangle$ and $\mathbf{v} = \langle -1, 2, 1 \rangle$?

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 1 \\ -1 & 2 & 1 \end{vmatrix} = \langle 1, -1, 3 \rangle$$

$$A = \| \langle 1, -1, 3 \rangle \| = \sqrt{1+1+9} = \boxed{\sqrt{11}}$$

(7 pts) 4. Given the surface defined by the graph of $z = x - y^2 + 3$, what is the equation of the y, z-trace?

y, z-trace is when $x = 0$: $\boxed{z = -y^2 + 3}$

(7 pts) 5. If $\mathbf{T}(t) = (1/\sqrt{2})\,\langle \cos t, 1, \sin t\rangle$, what is $\mathbf{B}(t)$?

$$\vec{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$$

$$\vec{T}'(t) = \frac{1}{\sqrt{2}} < -\sin t, 0, \cos t >$$

$$\|\vec{T}'(t)\| = \frac{1}{\sqrt{2}}\sqrt{\sin^2 t + \cos^2 t} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \vec{N} = < -\sin t, 0, \cos t >$$

Then $\vec{B} = \vec{T} \times \vec{N} = \frac{1}{\sqrt{2}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos t & 1 & \sin t \\ -\sin t & 0 & \cos t \end{vmatrix}$

$$= \boxed{\frac{1}{\sqrt{2}} < \cos t, -1, \sin t >}$$