Spring 2015

MAC2313 Test 1 A

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 1 & 0 & -3 \end{vmatrix} = -3\hat{i} + 6\hat{j} - \hat{k}$$

$\langle -3, 6, -1\rangle \cdot \langle 0, 1, 1\rangle$

$= 6 - 1 = 5$

(5 pts) 1. What is the value of $(\langle 2,1,0\rangle \times \langle 1,0,-3\rangle) \cdot \langle 0,1,1\rangle$?

A. -2 B. 0 C. 2 D. 5 E. none of the above

(5 pts) 2. Let **u** and **w** be vectors in R^3 such that $\|\mathbf{u}\| = 3$, $\|\mathbf{w}\| = 2$ and the angle between the two vectors is $\pi/2$; what is the value of $\|3\mathbf{u} - 4\mathbf{w}\|$?

$\|3u - 4w\|^2 = (3u-4w)\cdot(3u-4w) = 9\|u\|^2 - 24u\cdot w + 16\|w\|^2 = 9(9) - 24(3)(2)\overset{0}{\cos\frac{\pi}{2}} + 16(4)$

$= 81 + 64 = 145$

A. $\sqrt{59}$ B. $5\sqrt{5}$ C. $\sqrt{145}$ D. $\sqrt{43}$ E. $2\sqrt{17}$

(5 pts) 3. If **u** and **v** are vectors in R^3 and c is a scalar, how many of the following are true?

i. $\mathbf{v} \cdot (c\mathbf{u}) = c(\mathbf{u} \cdot \mathbf{v})$ ✓

ii. $\|c\mathbf{v}\| = |c|\,\|\mathbf{v}\|$ ✓

iii. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$ ✓

iv. $\mathbf{v} \times (c\mathbf{u}) = (-c\mathbf{u}) \times \mathbf{v}$ ✓

$-(cu) \times v$

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 4. Is the following equality true or false: $|\mathbf{v}| = \dfrac{ds}{dt}$?

$v(t) = \dfrac{ds}{dt}$

A. True B. False

(5 pts) 5. Let $\mathbf{a} = \langle 0, 6, 2\rangle$ and $\mathbf{b} = \langle 0, 2, 4\rangle$, then the component of $\mathbf{a}$ parallel to $\mathbf{b}$ is given by?

A. $\langle 0, -1, 3\rangle$ B. $\langle 0, 3, 1\rangle$ C. $\langle 0, 4, 8\rangle$ $\boxed{\text{D. } \langle 0, 2, 4\rangle}$ E. none of the above

$a_{\parallel} = \left(\frac{a \cdot b}{b \cdot b}\right) b = \frac{12+8}{4+16} b = \frac{20}{20} b = b = \langle 0, 2, 4\rangle$

(5 pts) 6. If $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the standard unit vectors and c is a nonzero scalar, then how many of the following are true?

i. $|\mathbf{j} \times (\mathbf{j} \times \mathbf{k})| = 1$ ✓

$j \times (j \times k) = j \times i = -k, \; |-k| = 1$

ii. $|c\mathbf{i} \cdot (\mathbf{j} + c\mathbf{k})| > 0$ ✗

$ci \cdot (j + ck) = ci \cdot j + c^2 i \cdot k = 0$

iii. $\mathbf{j}$ is parallel to $\mathbf{i} \times \mathbf{k} = -j$

✓

iv. $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ ✓

A. 0 B. 1 C. 2 $\boxed{\text{D. } 3}$ E. 4

(5 pts) 7. Which of the following points lie on the curve of intersection of the paraboloid $z = 2x^2 + 2y^2 - 5$ and the plane $y = 1$?

A. $(2, 1, 5)$ B. $(0, 1, -2)$ C. $(1, 1, 1)$ D. $(-1, 1, 1)$ E. none of the above

$z = 2x^2 + 2 - 5$

$z = 2x^2 - 3$

(5 pts) 8. The domain of the function $\mathbf{r}(t) = \left\langle \ln t, \sqrt{1 - e^t}, \cos(t^2 - 1) \right\rangle$ is:

$t > 0 \quad 1 \geq e^t \quad t \leq 0$

A. $(0, 1) \cup (1, \infty)$ B. $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$ C. $[0, 1) \cup (1, \infty)$ D. $(1, \infty)$

E. none of the above

$\{0\}$

(5 pts) 9. If $\mathbf{u}$ and $\mathbf{v}$ be vectors in R^3 such that $|\mathbf{u}| > 1$ and $|\mathbf{v}| > 1$, then $\mathbf{u} \cdot \mathbf{v} > 0$.

A. True B. False

Ctrex: $2i, 2j$ have magnitude greater than 1; but $2i \cdot 2j = 0$

(5 pts) 10. If $\mathbf{r}(t) = \langle 6t^2, -5, t^3 \rangle$, then the length of the curve on the interval $t \in [0, 3]$ is equal to:

$v(t) = \langle 12t, 0, 3t^2 \rangle$

A. 88 B. 102 C. 121 D. 144 E. none of the above

$$L = \int_0^3 \sqrt{144t^2 + 9t^4}\, dt = \int_0^3 t\sqrt{144 + 9t^2}\, dt \qquad u = 144 + 9t^2, \quad du = 18t\,dt$$

$$= \frac{1}{18}\int_{144}^{225} u^{1/2}\, du = \frac{1}{27} u^{3/2}\Big|_{144}^{225} = \frac{1}{27}\left(15^3 - 12^3\right) = \frac{\cancel{3^3}(5^3 - 4^3)}{\cancel{3^3}}$$

$$= 125 - 64 = 61$$

(5 pts) 11. The curvature of $\mathbf{r}(t) = \langle 2\cos t, \sin t, 3t\rangle$ at $t = 0$ is equal to:

$r' = \langle -2\sin t, \cos t, 3\rangle$, $r'' = \langle -2\cos t, -\sin t, 0\rangle$

$r'(0) = \langle 0, 1, 3\rangle$ $r''(0) = \langle -2, 0, 0\rangle$

A. 1/10 B. 1/4 C. 1/2 D. 1/5 E. none of the above

$\begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ -2 & 0 & 0 \end{vmatrix} = -6\hat{j} + 2\hat{k}$

$\|r' \times r''\| = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$\|r'\|^3 = \left(\sqrt{1+9}\right)^3 = \sqrt{10}^3 = 10\sqrt{10}$

$K = \frac{2\sqrt{10}}{10\sqrt{10}} = \boxed{\frac{1}{5}}$

(5 pts) 12. How many of the following are true?

i. $\mathbf{T} \times \mathbf{N} = \mathbf{B}$ ✓

ii. $\mathbf{B} \cdot \mathbf{N} = 0$ ✓

iii. $\kappa = |d\mathbf{T}/ds|$ ✓

iv. $\mathbf{N} = (d\mathbf{T}/ds)/|d\mathbf{T}/ds|$ ✓

$N = \frac{T'}{\|T'\|} = \frac{dT/dt}{vK} = \frac{\frac{dT}{ds} \cdot \cancel{\frac{ds}{dt}}}{\cancel{\frac{ds}{dt}} \cdot \|\frac{dT}{ds}\|} = \frac{\frac{dT}{ds}}{\|\frac{dT}{ds}\|}$

A. 0 B. 1 C. 2 D. 3 $\boxed{\text{E. 4}}$

☆ (5 pts) 13. The normal vector to the plane $-2x + 4y - 6z = 0$ is orthogonal to which of the following vectors:

$\langle -2, 4, -6\rangle \cdot \langle a, b, c\rangle = -2a + 4b - 6c = 0$

A. $\langle -1, 2, 3\rangle$ $\boxed{\text{B. } \langle -1, 1, 1\rangle}$ C. $\langle 6, 0, 2\rangle$ D. $\langle 1, -2, 3\rangle$ E. none of the above

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Name: ______________________ UF-ID: ________________ Section: _____________

(7 pts) 1. At a given point on a curve $\mathbf{N} = \left\langle 1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3} \right\rangle$, $\mathbf{T} = \left\langle 1/\sqrt{6}, 2/\sqrt{6}, 1/\sqrt{6} \right\rangle$, and $\mathbf{a} = \langle 2, 1, 2 \rangle$; find a_N and a_T.

(7 pts) 2. If $\mathbf{r}'(\mathrm{t}) = \langle 6\mathrm{t}^2 + 1, \pi\cos(\pi \mathrm{t}), 4\mathrm{e}^{\mathrm{t}} \rangle$ and $\mathbf{r}(0) = \langle 1, 0, 0 \rangle$, find $\mathbf{r}(\mathrm{t})$.

(7 pts) 3. If $\mathbf{T}(\mathrm{t}) = (1/\sqrt{3})\,\langle \sin \mathrm{t} + \cos \mathrm{t}, 1, \cos \mathrm{t} - \sin \mathrm{t} \rangle$, calculate $\mathbf{B}(\mathrm{t})$.

(7 pts) 4. A given ~~plane~~ Line is described by the system of equations $x = 2t + 7$, $y = -3t + 4$, and $z = 5 - t$; find the equation of a ~~plane~~ Line parallel to the given ~~plane~~ Line and which contains the point $(1, 1, 0)$.

(7 pts) 5. Give a parametric equation of the line segment connecting the points $(-1, 4, 0)$ and $(5, 2, -3)$.