MAC 2313

Exam 1

A. Sign your scantron sheet in the white area on the back.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Discussion Section number

C. Under "special codes", code in the test number 3, 3.

1 2 • 4 5 6 7 8 9 0

1 2 • 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code" encode A.

• B C D E

E. This test consists of multiple choice and free response questions. Make sure you check for errors. In the tear off sheet part you need to show full work in order to receive credit. The time allowed is 90 minutes.

F. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your free response exam and scantron to your proctor. **Be prepared to show your picture ID with a legible signature.**

MAC2313 Test 1 A Spring 2016

(5 pts) 1. How many of the following lines are parallel to the line $\mathbf{r}(t) = \langle 2+2t, -6t, -3+4t \rangle$?

$\vec{v} = \langle 2, -6, 4 \rangle$

$\div -2 = \langle -1, 3, -2 \rangle$

i. $\mathbf{r}(t) = \langle 3-2t, 2-6t, -3+4t \rangle$? ✗

ii. $\mathbf{r}(t) = \langle 2-2t, 6t, -3-4t \rangle$? ✓

iii. $\mathbf{r}(t) = \langle 3+2t, 2+6t, -3-4t \rangle$? ✗

iv. $\mathbf{r}(t) = \langle 5-t, -3t, 2+2t \rangle$? ✗

A. 0 B. 1 C. 2 D. 3 E. 4

?

3 true?

(5 pts) 2. How many of the following are necessarily true?

i. Two vectors are orthogonal if their dot product is equal to the zero ~~vector~~ scalar. ✗

ii. Two vectors are parallel if one vector is a scalar multiple of the other vector. ✓

iii. Two planes are orthogonal if the normal vectors to the planes are orthogonal. ✓

iv. An equation of the line segment connecting the points $(1,2,0)$ and $(-2,4,1)$ is given by $\mathbf{r}(t) = \langle 1-3t, 2+2t, t \rangle$, $t \in [0,1]$ ✗

$\vec{v} = \langle -3, 2, 1 \rangle$

$\langle 1-3t, 2+2t, t \rangle$

A. 0 B. 1 $\boxed{\text{C. 2}}$ D. 3 E. 4

(5 pts) 3. Given the vector $\mathbf{u} = \langle 2+c, -3c, 8+2c \rangle$, what should be the value of c so that $\mathbf{u}$ is orthogonal to the vector $\langle 2, 2, -1 \rangle$?

$\boxed{\text{A. } -2/3}$ B. 0 C. $-1/3$ D. $4/3$ E. none of the above

$2(2+c) + 2(-3c) - (8+2c) = 0$

$4 + \cancel{2c} - 6c - 8 - \cancel{2c} = 0$

$6c = -4$

$c = -2/3$

(5 pts) 4. Let $\mathbf{u}$ and $\mathbf{w}$ be vectors in R^3 such that $\|\mathbf{u}\| = 2$, $\|\mathbf{w}\| = 3$, and $\mathbf{u} \times \mathbf{w} = \langle 1, -2, 2 \rangle$. Assuming the angle between the vectors is less than or equal to $\pi/2$, this angle is given by:

$\|u \times w\| = \sqrt{1+4+4} = \sqrt{9} = 3 \qquad \|u\| \cdot \|w\| \sin\theta = 6\sin\theta = 3 \Rightarrow \sin\theta = 1/2 \Rightarrow \theta = \frac{\pi}{6}$

A. $\sin^{-1}(1/\sqrt{2})$ B. $\cos^{-1}(1/\sqrt{3})$ C. $\cos^{-1}(12/\sqrt{6})$ D. $\sin^{-1}(\sqrt{2})$ $\boxed{\text{E. } \pi/6}$

(5 pts) 5. How many of the following points lie in the plane containing the point $(-2, 0, 3)$ and with a normal vector $\langle 0, 1, 1 \rangle$?

$\boxed{y + z = 3}$

i. $(0, 3, 0)$ ✓

ii. $(1, -2, 1)$ ✗

iii. $(1, 2, 1)$ ✓

iv. $(2, -1, 0)$ ✗

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 6. For a curve $\mathbf{r}(t) = \langle 3 + 2t, t^2, t^2 + t \rangle$, $t \in (-\infty, \infty)$, the curvature at $t = 1$ is given by:

$r' = \langle 2, 2t, 2t+1 \rangle, \quad r'' = \langle 0, 2, 2 \rangle \Rightarrow \vec{r}'(1) = \langle 2, 2, 3 \rangle$

A. $\sqrt{30}/(\sqrt{17})^3$ B. $\sqrt{24}/(\sqrt{22})^3$ C. $6/125$ D. $12/5^3$ $\boxed{\text{E. } 6/(\sqrt{17})^3}$

$\begin{vmatrix} i & j & k \\ 2 & 2 & 3 \\ 0 & 2 & 2 \end{vmatrix} = \langle -2, -4, 4 \rangle$

$\|r' \times r''\| = \sqrt{4+16+16} = \sqrt{36} = 6$

$\|r'\| = \sqrt{4+4+9} = \sqrt{17} \qquad \Rightarrow \kappa = \frac{6}{(\sqrt{17})^3}$

(5 pts) 7. Let $\mathbf{u} = \langle 0, 6, 2 \rangle$ and $\mathbf{v} = \langle 0, 2, 4 \rangle$, then $\mathbf{u}_\perp$ is given by?

A. $\langle 0, -1, 3 \rangle$ B. $\langle 0, 3, 1 \rangle$ C. $\langle 0, 4, 8 \rangle$ D. $\langle 0, 2, 4 \rangle$ $\boxed{\text{E. none of the above}}$

$u_{\parallel} = \frac{12+8}{4+16} v = \frac{20}{20} v = v = \langle 0, 2, 4 \rangle$

$u_\perp = \langle 0, 6, 2 \rangle - \langle 0, 2, 4 \rangle = \langle 0, 4, -2 \rangle$

(5 pts) 8. A parametrized curve defined for all t has a unit tangent vector given by $\mathbf{T}(t) = (1/\sqrt{10})\langle -\cos 2t, -3, \sin 2t\rangle$. The Principal Unit Normal vector to the curve is given by:

A. $(1/2\sqrt{10})\langle \sin 2t, 0, -\cos 2t\rangle$ B. $(1/\sqrt{10})\langle -\sin 2t, 0, \cos 2t\rangle$ $\boxed{\text{C. } \langle \sin 2t, 0, \cos 2t\rangle}$

D. $(1/2)\langle -\sin 2t, 0, -\cos 2t\rangle$ E. none of the above

$T' = \frac{1}{\sqrt{10}}\langle 2\sin 2t, 0, 2\cos 2t\rangle$

$\|T'\| = \frac{1}{\sqrt{10}}\sqrt{4} = \frac{2}{\sqrt{10}}$

$\vec{N} = \frac{\cancel{\sqrt{10}}}{\cancel{2}} \cdot \frac{1}{\cancel{\sqrt{10}}}\langle \cancel{2}\sin 2t, 0, \cancel{2}\cos 2t\rangle$
$= \langle \sin 2t, 0, \cos 2t\rangle$

(5 pts) 9. Given the plane $2x - 3y - 5z = 13$, how many of the following are true?

$\vec{n} = \langle 2, -3, -5\rangle$

i. The plane is orthogonal to the plane $4x + 6y + 2z = 13$.

$\vec{n} = \langle 4, 6, 2\rangle \cdot \langle 2, -3, -5\rangle = 8 - 18 - 10 = -20$ ⊗

ii. The plane is parallel to the plane $-2x + 3y - 5z = 0$. ⊗

$\vec{n} = \langle -2, 3, -5\rangle$

iii. A normal vector to the plane is $\langle 2, -3, -5\rangle$. ✓

iv. The plane is parallel to the y, z-coordinate plane. $x = 0$ has $n = \langle 1, 0, 0\rangle$ ⊗

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 10. At a certain point on a curve in R^3. $\mathbf{T} = (1/\sqrt{3})\langle 1, 1, -1\rangle$ and $\mathbf{N} = (1/\sqrt{6})\langle 1, 1, 2\rangle$. Which of the following vectors is equal to $\mathbf{B}$ at that point?

A. $(1/\sqrt{3})\langle 1, -1, 1\rangle$ B. $(1/\sqrt{3})\langle -1, 1, -1\rangle$ C. $(1/\sqrt{2})\langle 1, -1, 0\rangle$ D. $(1/\sqrt{18})\langle 3, -3, 0\rangle$

E. none of the above

$$T \times N = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{6}} \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 1 & 2 \end{vmatrix} = \frac{1}{\sqrt{18}}\langle 3, -3, 0\rangle$$

(5 pts) 11. The domain of the function $\mathbf{r}(t) = \left\langle 1/(t^2-1), \sqrt{4+t^2}, t/e^{2t} \right\rangle$ is:

A. $(0,1)\cup(1,\infty)$ B. $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$ C. $[0,1)\cup(1,\infty)$

D. $(-4,-1)\cup(-1,1)\cup(1,\infty)$ E. none of the above

$t \neq \pm 1$, $t^2+4 \geq 0$ always

$e^{2t} \neq 0$ never

$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

(5 pts) 12. There exists vectors **u** and **v** in R^3 such that $|\mathbf{u}| \neq 0$ and $|\mathbf{v}| \neq 0$ but $|\mathbf{u}\times\mathbf{v}| = 0$.

$\hat{i} \times 2\hat{i} = \vec{0} \Rightarrow \|\hat{i}\times 2\hat{i}\| = 0$

A. True B. False

(5 pts) 13. Given a plane defined by the three points $(1,1,1)$, $(-4,0,1)$, and $(-3,1,0)$, a vector normal to the plane is given by: $PQ = \langle -5,-1,0\rangle$ $PR = \langle -4,0,-1\rangle$ $\vec{n} = \begin{vmatrix} i & j & k \\ -5 & -1 & 0 \\ -4 & 0 & -1 \end{vmatrix} = \langle 1,-5,-4\rangle$

A. $\langle -1,0,-4\rangle$ B. $\langle 0,-1,-4\rangle$ C. $\langle 1,-5,-4\rangle$ D. $\langle -1,-5,0\rangle$ E. $\langle 1,-5,0\rangle$

Bonus (5 pts) 14. Assuming that the vectors **T**, **N**, and **B** exist at all points on a given curve, what is the value of $-2\mathbf{N}\cdot(5\mathbf{T}-4\mathbf{B}+\mathbf{N})$? $-10\underset{0}{(\cancel{N\cdot T})} + 8\underset{0}{(\cancel{N\cdot B})} - 2\underset{1}{(\cancel{N\cdot N})} = -2$

A. 0 B. 2 C. -4 D. -2 E. 4

MAC2313 Test 1 A

Name: ______________________ UF-ID: ________________ Section: _____________

(6 pts) 1. Give a parametric equation (including the domain) of the line segment connecting the points $(1, -2, 0)$ and $(0, 2, -3)$.

(6 pts) 2. Fill in the blanks:

a. A _________________ of a surface is the set of all points at which the surface intersects a plane that is parallel to one of the coordinate planes.

b. The acceleration vector for a trajectory lies in the plane formed by the vectors ______ and ______.

c. For a curve in R^3, the vector ______ points in the direction in which the curve is turning.

d. _________________ is the magnitude of the rate of change of $\mathbf{T}$ with respect to arclength along a given curve.

e. Given a curve C in a plane P, and a line l not in P, a _________________ is the surface consisting of all lines parallel to l and passing through C.

(8 pts) 3. If a trajectory for an object is given by $\mathbf{r}(t) = \langle -\cos 3t, \sin 2t, e^{3t} \rangle$, find the acceleration vector for the object when $t = \pi$.

(8 pts) 4. If $\mathbf{r}'(t) = \langle 6t^2, -2, 4t + 1 \rangle$ and $\mathbf{r}(1) = \langle 1, -1, 0 \rangle$, find $|\mathbf{r}(2)|$.

(7 pts) 5. A curve is given parametrically by $r(t) = < \cos t, \sin t, t^{3/2}>$ with $0 \le t \le 2$. Compute the length of the curve.