(5 pts) 1. The equation of the tangent plane to surface $25x^2 + 4y^2 + 4z^2 = 100$ at the point $(0, 4, 3)$ is:

A. $32y + 24z = 200$  
B. $8x + 24y + 32z = 200$  
C. $8x + 32y + 24z = 200$  
D. $24y + 32z = 200$  
E. none of the above

(5 pts) 2. If $w$ is a function of the variables $x$, $y$, and $z$ and each of the variables $x$, $y$, and $z$ is a function of the variables $s$ and $t$, then which of the following is true?

A. $\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}$  
B. $\frac{dw}{ds} = \frac{\partial w}{\partial x} \frac{dx}{ds} + \frac{\partial w}{\partial y} \frac{dy}{ds} + \frac{\partial w}{\partial z} \frac{dz}{ds}$

C. $\frac{dw}{ds} = \frac{\partial w}{\partial x} \frac{dx}{ds} + \frac{\partial w}{\partial y} \frac{dy}{ds} + \frac{\partial w}{\partial z} \frac{dz}{ds}$  
B. $\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{dx}{ds} + \frac{\partial w}{\partial y} \frac{dy}{ds} + \frac{\partial w}{\partial z} \frac{dz}{ds}$

E. none of the above

(5 pts) 3. The derivative $\frac{\partial^2}{\partial x \partial y} (y + y^2 e^x)$ is equal to:

A. $ye^x$  
B. $2ye^x$  
C. $x + y + ye^x$  
D. $2x + 2y + 2ye^x$  
E. $1 + 2ye^x$

(5 pts) 4. If $\mathbf{u} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$ and $f(x, y) = x^2 + y$, then $D_uf(1, 0)$ is equal to:

A. $2/\sqrt{13}$  
B. $3/\sqrt{13}$  
C. $5/\sqrt{13}$  
D. $6/\sqrt{13}$  
E. $7/\sqrt{13}$
(5 pts) 5. Which of the following are critical points of the function \( f(x, y) = x^2 + 2y^2 - 4xy - 6x \)?

I. \((3, 3)\)

II. \((-3, -3)\)

III. \((0, 0)\)

IV. \((1, 1)\)

A. only I  B. only II  C. only III and IV  D. only I and II  E. only I, III, and IV

(5 pts) 6. The equation of the tangent plane to the surface \( z = \cos(xy) \) at the point \((2, \pi, 1)\) is given by:

A. \( L(x, y) = \pi x + 2y \)  
B. \( L(x, y) = \pi x + 2y - 2\pi \)  
C. \( L(x, y) = \pi x + 2y + 2\pi \)

D. \( L(x, y) = \pi x + 2y + 4\pi \)  
E. none of the above

(5 pts) 7. Given the function \( g(x, y, z) = \sqrt{16 - x^2 - y^2 - z^2} \), which of the following are true?

I. \( D = \{ (x, y, z) \mid 16 \geq x^2 + y^2 + z^2 \} \)

II. \( R = [0, 16] \)

III. The level surfaces are spheres.

IV. The graph of the function \( g(x, y, z) \) lies in \( R^4 \).

A. only I  B. only III  C. only II and IV  D. only I and II  E. only I, III, and IV
(5 pts) 8. If \( f(x, y) = (2xy + 3y^2)^3 \), then \( f_x(-1, 1) \) is equal to:

A. \(-4\)  
B. 4  
C. 6  
D. 8  
E. 12

(5 pts) 9. Let \((a, b)\) be a point in the domain of the real-valued function \( f(x, y) \) and let \( L \in \mathbb{R} \); which of the following are true?

I. If \( f(x, y) \) is continuous at \((a, b)\), then \( \lim_{(x,y) \to (a,b)} f(x, y) \) exists.

II. If \( \lim_{(x,y) \to (a,b)} f(x, y) \) exists, then \( f(x, y) \) is continuous at \((a, b)\).

III. If \( f(x, y) \) is continuous at \((a, b)\), then \( \lim_{(x,y) \to (a,b)} f(x, y) = f(a, b) \).

IV. If \( \lim_{(x,y) \to (a,b)} f(x, y) = L \), then \( \lim_{(x,y) \to (a,b)} \sqrt{f(x, y)} = \sqrt{L} \).

A. only I  
B. only II  
C. only I and II  
D. only II and IV  
E. I and III

(5 pts) 10. For \( f(x, y) = x^3 + y^3 - 12xy \), the critical point \((4, 4)\) is:

A. a local max  
B. a local min  
C. a saddle point  
D. a boundary point  
E. the second derivative test fails at this point

(5 pts) 11. To find the point on the surface \( z = x^2 - y^2 \) closest to the point \((1, 2, 3)\), which of the following functions should be optimized?

A. \( \sqrt{x^2 + y^2 + z^2} \)  
B. \( \sqrt{(x+1)^2 + (y+2)^2 + (z+3)^2} \)  
C. \( \sqrt{x^2 + y^2 + (x^2 - y^2)^2} \)

D. \( (x-1)^2 + (y-2)^2 + (x^2 - y^2 - 3)^2 \)  
E. none of the above
(5 pts) 12. Which of the following is a vector orthogonal to the tangent line to the curve \(x^2 - 4xy - y^2 = 4\) at the point \((5, 1)\)?

A. \((-6, 2)\)  
B. \((1, 3)\)  
C. \((-1, 3)\)  
D. \((6, -22)\)  
E. \((-11, 3)\)

(5 pts) 13. Let \(f(x, y, z) = x^2 + yz + \cos(\pi z)\), \(x = t\), \(y = \cos t\), and \(z = e^t\). Calculate the value of derivative of the \(f(x, y, z)\) with respect to \(t\) when \(t = 0\).

A. 0  
B. 1  
C. -1  
D. 1 + \pi  
E. 1 - \pi

(5 pts) 14. Bonus. Let \(f(x, y)\) function which is differentiable at the point \((a, b)\) and let \(\mathbf{u}\) be a nonzero vector in \(\mathbb{R}^2\). Which of the following are true?

I. The equation of the tangent plane to the surface at \((a, b, f(a, b))\) is given by \(L(x, y) = f_y(a, b)(x - a) + f_x(a, b)(y - b)\).

II. \(D_u f(a, b) = \langle f_x(a, b), f_y(a, b) \rangle \cdot \mathbf{u}\)

III. The max rate of change of \(f(x, y)\) at \((a, b)\) is in the direction of \(\nabla f(a, b)\).

IV. If \(\nabla f(a, b) = 0\), then \((a, b)\) is a critical point of \(f(x, y)\).

A. I and III  
B. II and III  
C. I and IV  
D. III and IV  
E. II and IV
(6 pts) 1. Calculate the following limit:

$$\lim_{(x,y) \to (2,2)} \frac{x^2 + y^2}{x - y}$$

(6 pts) 2. Given $f(x, y) = 2x^2 + y^2$, use the value of the function at the point $(2, 1)$ and differentials to approximate $f(1.95, 1.1)$. 
(7 pts) 3. Use the technique of implicit differentiation discussed in class to calculate \( \frac{dy}{dx} \) given
\[ \sqrt{x^2 y^2 + 3y^4} = 2e^3. \]

(6 pts) 4. Given \( f(x, y) = x \sin y + x^2 \), at the point \((1, \pi)\) find a vector which points in the direction of no rate of change of the function.
5. Given \( f(x, y) = (x^2 - 1)(y^2 - 4), \) \( f_x(x, y) = 2x(y^2 - 4), \) and \( f_y(x, y) = 2y(x^2 - 1), \) find the max and min values of \( f(x, y) \) on the rectangular region bounded by the lines \( x = 1, \) \( x = -1, \) \( y = 2, \) and \( y = -2. \)