MAC2313 Test 2A

(5 pts) 1. The equation of the tangent plane to surface $25x^2 + 4y^2 + 4z^2 = 100$ at the point $(0, 4, 3)$ is:

A. $32y + 24z = 200$ B. $8x + 24y + 32z = 200$ C. $8x + 32y + 24z = 200$

D. $24y + 32z = 200$ E. none of the above

(5 pts) 2. If w is a function of the variables x, y, and z and each of the variables x, y, and z is a function of the variables s and t, then which of the following is true?

A. $\frac{\partial \mathrm{w}}{\partial \mathrm{s}} = \frac{\partial \mathrm{w}}{\partial \mathrm{x}}\frac{\partial \mathrm{x}}{\partial \mathrm{s}} + \frac{\partial \mathrm{w}}{\partial \mathrm{y}}\frac{\partial \mathrm{y}}{\partial \mathrm{s}} + \frac{\partial \mathrm{w}}{\partial \mathrm{z}}\frac{\partial \mathrm{z}}{\partial \mathrm{s}}$ B. $\frac{\mathrm{dw}}{\mathrm{ds}} = \frac{\partial \mathrm{w}}{\partial \mathrm{x}}\frac{\mathrm{dx}}{\mathrm{ds}} + \frac{\partial \mathrm{w}}{\partial \mathrm{y}}\frac{\mathrm{dy}}{\mathrm{ds}} + \frac{\partial \mathrm{w}}{\partial \mathrm{z}}\frac{\mathrm{dz}}{\mathrm{ds}}$

C. $\frac{\mathrm{dw}}{\mathrm{ds}} = \frac{\partial \mathrm{w}}{\partial \mathrm{x}}\frac{\partial \mathrm{x}}{\partial \mathrm{s}} + \frac{\partial \mathrm{w}}{\partial \mathrm{y}}\frac{\partial \mathrm{y}}{\partial \mathrm{s}} + \frac{\partial \mathrm{w}}{\partial \mathrm{z}}\frac{\partial \mathrm{z}}{\partial \mathrm{s}}$ B. $\frac{\partial \mathrm{w}}{\partial \mathrm{s}} = \frac{\partial \mathrm{w}}{\partial \mathrm{x}}\frac{\mathrm{dx}}{\mathrm{ds}} + \frac{\partial \mathrm{w}}{\partial \mathrm{y}}\frac{\mathrm{dy}}{\mathrm{ds}} + \frac{\partial \mathrm{w}}{\partial \mathrm{z}}\frac{\mathrm{dz}}{\mathrm{ds}}$

E. none of the above

(5 pts) 3. The derivative $\frac{\partial^2}{\partial x \partial y}\,(y + y^2 e^x)$ is equal to:

A. ye^x B. $2ye^x$ C. $x + y + ye^x$ D. $2x + 2y + 2ye^x$ E. $1 + 2ye^x$

(5 pts) 4. If $\mathbf{u} = \left\langle 2/\sqrt{13},\ 3/\sqrt{13} \right\rangle$ and $f(x, y) = x^2 + y$, then $\mathrm{D}_{\mathbf{u}}\mathrm{f}(1, 0)$ is equal to:

A. $2/\sqrt{13}$ B. $3/\sqrt{13}$ C. $5/\sqrt{13}$ D. $6/\sqrt{13}$ E. $7/\sqrt{13}$

(5 pts) 5. Which of the following are critical points of the function $f(x, y) = x^2 + 2y^2 - 4xy - 6x$?

I. $(3, 3)$

II. $(-3, -3)$

III. $(0, 0)$

IV. $(1, 1)$

A. only I B. only II C. only III and IV D. only I and II E. only I, III, and IV

(5 pts) 6. The equation of the tangent plane to the surface $z = \cos(xy)$ at the point $(2, \pi, 1)$ is given by:

A. $L(x, y) = \pi x + 2y$ B. $L(x, y) = \pi x + 2y - 2\pi$ C. $L(x, y) = \pi x + 2y + 2\pi$

D. $L(x, y) = \pi x + 2y + 4\pi$ E. none of the above

(5 pts) 7. Given the function $g(x, y, z) = \sqrt{16 - x^2 - y^2 - z^2}$, which of the following are true?

I. D $= \{\ (x, y, z) \mid 16 \geq x^2 + y^2 + z^2\ \}$

II. R $= [0, 16]$

III. The level surfaces are spheres.

IV. The graph of the function $g(x, y, z)$ lies in R^4.

A. only I B. only III C. only II and IV D. only I and II E. only I, III, and IV

(5 pts) 8. If $f(x,y) = (2xy + 3y^2)^3$, then $f_x(-1,1)$ is equal to:

A. -4 B. 4 C. 6 D. 8 E. 12

(5 pts) 9. Let (a,b) be a point in the domain of the real-valued function $f(x,y)$ and let $L \in R$; which of the following are true?

I. If $f(x,y)$ is continuous at (a,b), then $\lim_{(x,y)\to(a,b)} f(x,y)$ exists.

II. If $\lim_{(x,y)\to(a,b)} f(x,y)$ exists, then $f(x,y)$ is continuous at (a,b).

III. If $f(x,y)$ is continuous at (a,b), then $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

IV. If $\lim_{(x,y)\to(a,b)} f(x,y) = L$, then $\lim_{(x,y)\to(a,b)} \sqrt{f(x,y)} = \sqrt{L}$.

A. only I B. only II C. only I and II D. only II and IV E. I and III

(5 pts) 10. For $f(x,y) = x^3 + y^3 - 12xy$, the critical point $(4,4)$ is:

A. a local max B. a local min C. a saddle point D. a boundary point

E. the second derivative test fails at this point

(5 pts) 11. To find the point on the surface $z = x^2 - y^2$ closest to the point $(1,2,3)$, which of the following functions should be optimized?

A. $\sqrt{x^2+y^2+z^2}$ B. $\sqrt{(x+1)^2+(y+2)^2+(z+3)^2}$ C. $\sqrt{x^2+y^2+(x^2-y^2)^2}$

D. $(x-1)^2+(y-2)^2+(x^2-y^2-3)^2$ E. none of the above

(5 pts) 12. Which of the following is a vector orthogonal to the tangent line to the curve $x^2 - 4xy - y^2 = 4$ at the point $(5, 1)$?

A. $\langle -6, 2\rangle$ B. $\langle 1, 3\rangle$ C. $\langle -1, 3\rangle$ D. $\langle 6, -22\rangle$ E. $\langle -11, 3\rangle$

(5 pts) 13. Let $f(x, y, z) = x^2 + yz + \cos(\pi z)$, $x = t$, $y = \cos t$, and $z = e^t$. Calculate the value of derivative of the $f(x, y, z)$ with respect to t when $t = 0$.

A. 0 B. 1 C. -1 D. $1 + \pi$ E. $1 - \pi$

(5 pts) 14. Bonus. Let $f(x, y)$ function which is differentiable at the point (a, b) and let $\mathbf{u}$ be a nonzero vector in R^2. Which of the following are true?

I. The equation of the tangent plane to the surface at $(a, b, f(a, b))$ is given by $L(x, y) = f_y(a, b)(x - a) + f_x(a, b)(y - b)$.

II. $D_{\mathbf{u}}f(a, b) = \langle f_x(a, b), f_y(a, b)\rangle \cdot \mathbf{u}$

III. The max rate of change of $f(x, y)$ at (a, b) is in the direction of $\nabla f(a, b)$.

IV. If $\nabla f(a, b) = \mathbf{0}$, then (a, b) is a critical point of $f(x, y)$.

A. I and III B. II and III C. I and IV D. III and IV E. II and IV

MAC2313 Test 2A

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(6 pts) 1. Calculate the following limit:

$$\lim_{(x,y)\to(2,2)} \frac{x^2+y^2}{x-y}.$$

(6 pts) 2. Given $f(x,y) = 2x^2 + y^2$, use the value of the function at the point $(2,1)$ and differentials to approximate $f(1.95, 1.1)$.

(7 pts) 3. Use the technique of implicit differentiation discussed in class to calculate $\dfrac{dy}{dx}$ given $\sqrt{x^2y^2 + 3y^4} = 2e^3$.

(6 pts) 4. Given $f(x, y) = x \sin y + x^2$, at the point $(1, \pi)$ find a vector which points in the direction of no rate of change of the function.

(10 pts) 5. Given $f(x,y) = (x^2-1)(y^2-4)$, $f_x(x,y) = 2x(y^2-4)$, and $f_y(x,y) = 2y(x^2-1)$, find the max and min values of $f(x,y)$ on the rectangular region bounded by the lines $x = 1$, $x = -1$, $y = 2$, and $y = -2$.