(5 pts) 1. If the function $g(x,y,z)$ is integrated over the cylindrical solid bounded by $x^2+y^2=3,\ z=1$, and $z=7$, the correct integral in Cartesian coordinates is given by:

A. $\int_1^7 \int_{-\sqrt{3}}^{\sqrt{3}} \int_0^{\sqrt{3-x^2}}\ g(x,y,z)\ dy\ dx\ dz$

B. $\int_1^7 \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\ g(x,y,z)\ dy\ dx\ dz$

C. $\int_1^7 \int_0^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\ g(x,y,z)\ dy\ dx\ dz$

D. $\int_1^7 \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\ g(x,y,z)\ dy\ dx\ dz$

E. $\int_1^7 \int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\ g(x,y,z)\ dy\ dx\ dz$

(5 pts) 2. Let R be a bounded region in the x,y-plane, D a bounded solid in R^3, and let the functions $f(x,y)$, $g(x,y)$, and $h(x,y,z)$ be continuous. Which of the following are true?

I. If $f(x,y) \geq g(x,y)$ on R, then the volume of the solid between these surfaces over R is equal to $\int\int_R\ g(x,y) - f(x,y)\ dA$.

II. The area of R is equal to $\int\int_R\ 1\ dA$.

III. The volume of D is equal to $\int\int\int_D\ 1\ dV$.

IV. The average value of $h(x,y,z)$ on D is equal to $\int\int\int_D\ h(x,y,z)\ dV$.

A. only I B. only II and III C. only I and IV D. only I, II, and III E. I, II, III, and IV

(5 pts) 3. Let $R = \{\ (r,\theta)\ |\ 1 \leq r \leq 2,\ (\pi/3) \leq \theta \leq (\pi/2)\ \}$; if the function $f(x,y) = 12y$ is integrated over R, the value of the integral is given by:

A. 0 B. 7 C. 12 D. 14 E. -12

(5 pts) 4. Let $D = \{ \ (x, y, z) \mid x^2 + y^2 \le c, \ x \ge 0, \ y \ge 0, \ a \le z \le b \ \}$; an integral which gives the volume of D is:

A. $\int_a^b \int_0^\pi \int_0^c \ r \ dr \ d\theta \ dz$

B. $\int_a^b \int_0^{\pi/2} \int_0^{\sqrt{c}} \ 1 \ dr \ d\theta \ dz$

C. $\int_a^b \int_0^{2\pi} \int_0^c \ r \ dr \ d\theta \ dz$

D. $\int_a^b \int_0^{\pi/2} \int_0^{\sqrt{c}} \ r \ dr \ d\theta \ dz$

E. $\int_a^b \int_{-\pi/2}^{\pi/2} \int_0^{\sqrt{c}} \ 1 \ dr \ d\theta \ dz$

(5 pts) 5. Let $D = \{ \ (\rho, \phi, \theta) \mid 0 \le \rho \le 5, \ 0 \le \phi \le (\pi/2), \ (\pi) \le \theta \le (3\pi/2) \ \}$; which of the following describes the same solid?

A. $\{ \ (x, y, z) \mid 0 \le z \le \sqrt{25 - x^2 - y^2}, \ -\sqrt{25 - x^2} \le y \le \sqrt{25 - x^2}, \ -5 \le x \le 0 \ \}$

B. $\{ \ (x, y, z) \mid 0 \le z \le \sqrt{25 - x^2 - y^2}, \ -\sqrt{25 - x^2} \le y \le \sqrt{25 - x^2}, \ 0 \le x \le 5 \ \}$

C. $\{ \ (x, y, z) \mid 0 \le z \le \sqrt{25 - x^2 - y^2}, \ 0 \le y \le \sqrt{25 - x^2}, \ -5 \le x \le 5 \ \}$

D. $\{ \ (x, y, z) \mid 0 \le z \le \sqrt{25 - x^2 - y^2}, \ -\sqrt{25 - x^2} \le y \le 0, \ -5 \le x \le 0 \ \}$

E. $\{ \ (x, y, z) \mid 0 \le z \le \sqrt{5 - x^2 - y^2}, \ -\sqrt{25 - x^2} \le y \le 0, \ 0 \le x \le 5 \ \}$

(5 pts) 6. Let T be a transformation defined by $x = rt\cos\theta, \ y = rt\sin\theta, \ z = r^2$; $\mathrm{J}(r, t, \theta)$ is given by:

A. $2r^3t$

B. $4r^2t$

C. $8r^3t$

D. $8r^2t$

E. none of the above

(5 pts) 7. Let $f(x, y, z)$ and $g(x, y, z)$ be differentiable functions in R^3; if the function $f(x, y, z)$ is to be optimized given the constraint $g(x, y, z) = c$, then which of the following sets of equations must be simultaneously solved?

I. $f(x, y, z) = \lambda g(x, y, z)$

II. $\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$

III. $g(x, y, z) = c$

IV. $\nabla g(x, y, z) = c$

A. I and III B. I and IV C. II and III D. II and IV E. I and II

(5 pts) 8. The integral $\int_2^3 \int_{\pi/2}^{\pi} \int_0^5 \; r^3 \; dr \; d\theta \; dz$ when transformed into Cartesian coordinates is equal to:

A. $\int_2^3 \int_{-5}^0 \int_0^{\sqrt{25-x^2}} \; x^2 + y^2 \; dy \; dx \; dz$

B. $\int_2^3 \int_0^5 \int_0^{\sqrt{25-x^2}} \; x + y \; dy \; dx \; dz$

C. $\int_2^3 \int_{-5}^0 \int_{-\sqrt{25-x^2}}^0 \; x^2 + y^2 \; dy \; dx \; dz$

D. $\int_2^3 \int_{-5}^5 \int_0^{\sqrt{25-x^2}} \; x^2 + y^2 \; dy \; dx \; dz$

E. $\int_2^3 \int_{-5}^0 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} \; x^2 + y^2 \; dy \; dx \; dz$

(5 pts) 9. The integral $\int_0^7 \int_0^{\sqrt{49-x^2}} \int_0^{\sqrt{49-x^2-y^2}} \; x \; dz \; dy \; dx$ when written in spherical coordinates is equal to:

A. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^7 \; \rho^3 \sin\theta \sin^2\phi \; d\rho \; d\phi \; d\theta$

B. $\int_0^{\pi} \int_0^{\pi/2} \int_0^7 \; \rho^3 \sin\theta \sin^2\phi \; d\rho \; d\phi \; d\theta$

C. $\int_0^{\pi} \int_0^{\pi/2} \int_0^7 \; \rho^3 \cos\theta \sin^2\phi \; d\rho \; d\phi \; d\theta$

D. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^7 \; \rho^3 \cos\theta \sin^2\phi \; d\rho \; d\phi \; d\theta$

E. $\int_0^{\pi} \int_0^{\pi/2} \int_0^7 \; \rho^3 \sin\theta \cos^2\phi \; d\rho \; d\phi \; d\theta$

(5 pts) 10. The integral $\int_0^\pi \int_1^2 3e^{2y} \cos 3x \; dy \; dx$ is equal to:

A. $e^4 + e^2$ B. $e^4 - 2e^2$ C. $2e^4 - e^2$ D. $e^4 - e^2$ E. none of the above

(5 pts) 11. If the function $f(x, y) = 6x$ is integrated over the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 3)$, the integral is equal to:

A. 0 B. 1 C. 2 D. 3 E. 4

(5 pts) 12. Given the integral $\int_0^2 \int_0^{x^3} f(x, y) \; dy \; dx$, if the order of integration is changed, the new integral is given by:

A. $\int_0^2 \int_{\sqrt[3]{y}}^2 f(x, y) \; dx \; dy$ B. $\int_0^8 \int_2^{\sqrt[3]{y}} f(x, y) \; dx \; dy$ C. $\int_0^4 \int_0^{\sqrt[3]{y}} f(x, y) \; dx \; dy$

D. $\int_0^8 \int_{\sqrt[3]{y}}^2 f(x, y) \; dx \; dy$ E. none of the above

(5 pts) 13. The average value of the function $f(x, y) = 4x + 3y^2$ on the rectangular region with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(0, 2)$ is given by:

A. 10 B. 11 C. 12 D. 13 E. 14

(5 pts) Bonus 14. Which of the following are true?

I. In spherical coordinates, $y = \rho \sin\theta \cos\phi$.

II. When converting from Cartesian to cylindrical coordinates, $z = r$.

III. In spherical coordinates, $z = \rho \cos\phi$.

IV. When converting from polar to Cartesian coordinates, $\theta = \tan^{-1}(y/x)$.

A. only II   B. only III   C. only I and III   D. only II and IV   E. I, II, III, and IV

(5 pts) Bonus 15. The volume which lies inside the cylinder $x^2 + y^2 = 4$ and which is bounded above by the sphere $x^2+y^2+z^2 = 5$, and below by the plane $z = 0$ is given by $\int_0^{2\pi} \int_0^4 \int_0^{\sqrt{5-r^2}} r\, dz\, dr\, d\theta$.

A. true   B. false

MAC2313 Test 3A

Name: ______________________ UF-ID: ________________ Section: _____________

(6 pts) 1. Set up but do not evaluate the integral in cylindrical coordinates used to integrate the function $f(x, y, z) = 2yz$ over the solid region bounded by the surfaces $z = 8 - 2x^2 - 2y^2$ and $z = 2$.

(6 pts) 2. Set up but do not evaluate the integral resulting from changing the order of integration in $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{y^2}^{2} \; x - y \; dx \; dy$.

(7 pts) 3. Give the integral in spherical coordinates used to determine the volume of the solid bounded above by $x^2 + y^2 + z^2 = 9$ and below by $z = \sqrt{x^2 + y^2}$. Do not evaluate this integral.

(6 pts) 4. Consider the set of transformations $(u, v) = (2x - 2y, 2y)$ and $(x, y) = \left(\frac{u}{2} + \frac{v}{2}, \frac{v}{2}\right)$ which are inverses of each other. Let D be the region in the x, y- plane bounded by the parallelogram with vertices $(0, 0)$, $(1, 0)$, $(2, 1)$, and $(1, 1)$ and let R be the region in the u, v- plane bounded by the square with vertices $(0, 0)$, $(2, 0)$, $(2, 2)$, and $(0, 2)$. Note that $J(x, y) = 4$ and $J(u, v) = 1/4$. Transform the integral $\int\int_D 16(x + y)\, dA$ into a double integral with respect to the variables u and v. Make sure you give the correct limits in the new integral. (you do not need to evaluate this integral)

(10 pts) 5. Use the Method of Lagrange Multipliers to find the max and min values of $f(x, y) = 6x - 12y$ on the circle $x^2 + y^2 = 9$.