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Quiz 1
You must show all work to receive full credit!!

Problem 1. (2 points) Determine whether the relation $y - \ln y = x^2 + 1$ is an implicit solution to the differential equation

$$\frac{dy}{dx} = \frac{2xy}{y-1}.$$

By implicit differentiation, $\frac{d}{dx}\left[y - \ln y\right] = \frac{d}{dx}\left[x^2+1\right]$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{y}\cdot\frac{dy}{dx} = 2x$$

$$\Rightarrow \frac{dy}{dx}\left(1-\frac{1}{y}\right) = \frac{dy}{dx}\left(\frac{y-1}{y}\right) = 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy}{y-1}. \checkmark$$

So the relation is an implicit solution.

Problem 2. (3 points) Solve the initial value problem

$$t^{-1}\frac{dy}{dt} = 2\cos^2 y, \quad y(0) = \frac{\pi}{4}.$$

Separating variables gives $\int \frac{1}{2}\sec^2 y\, dy = \int t\, dt$

$$\Rightarrow \frac{1}{2}\tan y = \frac{1}{2}t^2 + c$$

$$\Rightarrow \tan y = t^2 + C \Rightarrow y = \tan^{-1}(t^2 + c)$$

Initial condition: $y(0) = \tan^{-1}(c) = \frac{\pi}{4} \Rightarrow c = \tan\left(\frac{\pi}{4}\right) = 1.$

Thus, $\boxed{y(t) = \tan^{-1}(t^2+1)}$.