Quiz 1
You must show all work to receive full credit!!

Problem 1. (2 points) Determine whether the relation \( y - \ln y = x^2 + 1 \) is an implicit solution to the differential equation

\[
\frac{dy}{dx} = \frac{2xy}{y-1}.
\]

By implicit differentiation, \( \frac{d}{dx} [y - \ln y] = \frac{d}{dx} [x^2 + 1]\)

\[
\Rightarrow \frac{dy}{dx} - \frac{1}{y} \cdot \frac{dy}{dx} = 2x
\]

\[
\Rightarrow \frac{dy}{dx} \left( 1 - \frac{1}{y} \right) = \frac{dy}{dx} \left( \frac{y-1}{y} \right) = 2x
\]

\[
\Rightarrow \frac{dy}{dx} = \frac{2xy}{y-1}.
\]

So the relation is an implicit solution.

Problem 2. (3 points) Solve the initial value problem

\[
t^{-1} \frac{dy}{dt} = 2 \cos^2 y, \quad y(0) = \frac{\pi}{4}.
\]

Separating variables gives \( \int \frac{1}{2} \sec^2 y \, dy = \int t \, dt \)

\[
\Rightarrow \frac{1}{2} \tan y = \frac{1}{2} t^2 + C
\]

\[
\Rightarrow \tan y = t^2 + C \Rightarrow y = \tan^{-1}(t^2 + C)
\]

Initial condition: \( y(0) = \tan^{-1}(C) = \frac{\pi}{4} \Rightarrow C = \tan \left( \frac{\pi}{4} \right) = 1. \)

Thus, \( y(t) = \tan^{-1}(t^2 + 1). \)