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Quiz 1

You must give complete, mathematically correct proofs to receive full credit!!

Problem 1. (4 points) Determine whether the following element of D_3 is a rotation or a reflection, and justify your reasoning:

$$R_{120}R_{240}DVR_0D'R_{120}V.$$

Whether an element of D_n is a reflection or rotation depends only on the number of reflections: an even number will return the figure to its starting side while an odd number will flip it to the back side. Since the given element involves 4 reflections and 4 is even, the element must be a rotation.

Problem 2. (6 points) Let $S = \mathbb{Z} \setminus \{0\}$ and define the relation $a \sim b$ if $ab > 0$. Decide whether or not $\sim$ is an equivalence relation on S. If it is, give a proof of this and describe the equivalence classes. If not, state which properties are satisfied and which properties fail to hold.

Yes, it is an equivalence relation.

Proof. Let $a \in S = \mathbb{Z} \setminus \{0\}$. Then $a^2 > 0$ implies $a \sim a$, so $\sim$ is reflexive. Suppose that $a \sim b$. Then $ab > 0$ implies $ba > 0$, so $b \sim a$ and $\sim$ is symmetric. Finally, suppose that $a \sim b$ and $b \sim c$ for $a, b, c \in S$. Then $ab > 0$ and $bc > 0$. Since $ab > 0$, a and b must have the same sign, and similarly b and c must have the same sign. Therefore, a and c must have the same sign, so $ac > 0$ and $a \sim c$, showing that $\sim$ is transitive. Thus, $\sim$ is an equivalence relation. ▨

As implied in the proof, the equivalence classes are determined by the sign of the integer, so there are two distinct equivalence classes:

$$\overline{1} = \{1, 2, 3, \dots\} = \mathbb{Z}^+ \text{ and } \overline{-1} = \{-1, -2, -3, \dots\} = \mathbb{Z}^-.$$