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MAC 2313.3122
Cyr

Quiz 1
You must show all work to receive full credit!!

Problem 1. *(4 pts) Let* $\vec{v} = \langle 2, -4, 7 \rangle$.
(a) Find the unit vector $\vec{u}$ *which points in the same direction as* $\vec{v}$.

$$\|\vec{v}\| = \sqrt{2^2+(-4)^2+7^2} = \sqrt{4+16+49} = \sqrt{69}$$

$$\vec{u} = \frac{1}{\sqrt{69}}\vec{v} = \boxed{\left\langle \frac{2}{\sqrt{69}}, \frac{-4}{\sqrt{69}}, \frac{7}{\sqrt{69}} \right\rangle}$$

(b) Write the vector equation of the line passing through $(-5, 6, 1)$ *with direction vector* $\vec{v}$.

$$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$
$$= \langle -5, 6, 1 \rangle + t\langle 2, -4, 7 \rangle$$
$$= \boxed{\langle -5+2t, 6-4t, 1+7t \rangle}$$

Problem 2. *(6 pts) Find the orthogonal decomposition of* $\vec{u} = \langle 4, -1, 5 \rangle$ *with respect to* $\vec{v} = \langle 2, 1, 1 \rangle$. *(That is, write* $\vec{u}$ *as a sum of two vectors, one parallel to* $\vec{v}$ *and the other orthogonal to* $\vec{v}$.*)*

$$\vec{u}_{\parallel} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\frac{12}{6}\right)\vec{v} = 2\vec{v} = \langle 4, 2, 2 \rangle$$

$$\vec{u}\cdot\vec{v} = (4\cdot 2)+(-1\cdot 1)+(5\cdot 1) = 8-1+5 = 12$$
$$\vec{v}\cdot\vec{v} = \|\vec{v}\|^2 = 2^2+1^2+1^2 = 4+1+1 = 6$$

$$\vec{u}_{\perp} = \vec{u} - \vec{u}_{\parallel} = \langle 4, -1, 5 \rangle - \langle 4, 2, 2 \rangle = \langle 0, -3, 3 \rangle.$$

Thus, $\vec{u} = \vec{u}_{\parallel} + \vec{u}_{\perp} = \boxed{\langle 4, 2, 2 \rangle + \langle 0, -3, 3 \rangle}$