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MAC 2313.9256
Cyr

Quiz 1

You must show all work to receive full credit!!

Problem 1. Let $\mathbf{u} = \langle 2, -1, 3 \rangle$ and $\mathbf{v} = \langle -2, 3, 2 \rangle$.

(a) (3 pts) Find a vector parametrization for the line passing through the point $(1, 4, -6)$ in the direction of the vector $\mathbf{u} + \mathbf{v}$.

$$\vec{u} + \vec{v} = \langle 2, -1, 3 \rangle + \langle -2, 3, 2 \rangle = \langle 2-2, -1+3, 3+2 \rangle = \langle 0, 2, 5 \rangle$$

$$\text{Vector parametrization: } \vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

$$= \langle 1, 4, -6 \rangle + t \langle 0, 2, 5 \rangle$$

$$= \boxed{\langle 1, 4+2t, -6+5t \rangle}$$

(b) (2 pts) Determine whether the angle between \mathbf{u} and \mathbf{v} is acute or obtuse.

$$\vec{u} \cdot \vec{v} = \langle 2, -1, 3 \rangle \cdot \langle -2, 3, 2 \rangle$$

$$= 2(-2) + (-1)(3) + 3(2) = -4 - 3 + 6 = \underline{-1}$$

Since $\vec{u} \cdot \vec{v} = -1 < 0$, the angle between \vec{u} and \vec{v} is obtuse.

(c) (5 pts) Find the area of the parallelogram spanned by \mathbf{u} and \mathbf{v} .

$$\begin{aligned} \vec{u} \times \vec{v} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ -2 & 3 & 2 \end{vmatrix} = \langle -1 \cdot 2 - 3 \cdot 3, -2 \cdot 3 - 2 \cdot 2, 2 \cdot 3 - (-1)(-2) \rangle \\ &= \langle -2 - 9, -6 - 4, 6 - 2 \rangle = \underline{\langle -11, -10, 4 \rangle} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \|\vec{u} \times \vec{v}\| = \|\langle -11, -10, 4 \rangle\| = \sqrt{(-11)^2 + (-10)^2 + 4^2} \\ &= \sqrt{121 + 100 + 16} = \boxed{\sqrt{237}} \end{aligned}$$