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MAC 2313.6717
Cyr

Quiz 1
You must show all work to receive full credit!!

Problem 1. (5 points) Find the angle between the vectors $\mathbf{a} = \langle 1, 0, -1 \rangle$ and $\mathbf{b} = \langle -2, 2, 0 \rangle$.

$$\theta = \cos^{-1}\left(\frac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}\right) = \cos^{-1}\left(\frac{-2}{\sqrt{2}\cdot 2\sqrt{2}}\right) = \cos^{-1}\left(-\frac{2}{4}\right) = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\hat{a}\cdot\hat{b} = \langle 1,0,-1\rangle \cdot \langle -2,2,0\rangle = -2 + 0 + 0 = -2$$

$$|\hat{a}| = \sqrt{1^2+0^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$|\hat{b}| = \sqrt{(-2)^2+2^2+0^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$$

Problem 2. (5 points) Find the equation of the plane containing the point $(1, 0, 2)$ and the vectors $\langle 2, 3, 1\rangle$ and $\langle 4, 2, 5\rangle$.

Normal vector is cross product of vectors in the plane:

$$\hat{n} = \langle 2,3,1\rangle \times \langle 4,2,5\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 4 & 2 & 5 \end{vmatrix} = \langle 15-2, -(10-4), 4-12\rangle$$
$$= \langle 13, -6, -8\rangle$$

Plane equation is $\hat{n}\cdot\langle x,y,z\rangle = \hat{n}\cdot\langle x_0, y_0, z_0\rangle$

$$\langle 13,-6,-8\rangle\cdot\langle x,y,z\rangle = \langle 13,-6,-8\rangle\cdot\langle 1,0,2\rangle$$
$$= 13 + 0 - 16 = -3$$

$$\Rightarrow \boxed{13x - 6y - 8z = -3}$$