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MAC 2313.9722
Cyr

Quiz 1

You must show all work to receive full credit!!

Problem 1. (3 points) Find the vector projection of $\mathbf{b} = \langle 3, 1, 2 \rangle$ onto $\mathbf{a} = \langle 2, -2, 4 \rangle$.

$$\text{proj}_{\hat{\mathbf{a}}} \hat{\mathbf{b}} = \frac{\hat{\mathbf{a}} \cdot \hat{\mathbf{b}}}{\hat{\mathbf{a}} \cdot \hat{\mathbf{a}}} \hat{\mathbf{a}}$$

$$\hat{\mathbf{a}} \cdot \hat{\mathbf{b}} = \langle 2, -2, 4 \rangle \cdot \langle 3, 1, 2 \rangle = 6 - 2 + 8 = 12$$

$$\hat{\mathbf{a}} \cdot \hat{\mathbf{a}} = \langle 2, -2, 4 \rangle \cdot \langle 2, -2, 4 \rangle = 4 + 4 + 16 = 24$$

$$\Rightarrow \text{proj}_{\hat{\mathbf{a}}} \hat{\mathbf{b}} = \frac{12}{24} \hat{\mathbf{a}} = \frac{1}{2} \langle 2, -2, 4 \rangle = \boxed{\langle 1, -1, 2 \rangle}$$

Problem 2. (7 points) Find the equation of the plane passing through the points $(1, 0, 1)$, $(-2, 1, 3)$, and $(4, 2, 5)$.

Plane contains vectors $\vec{AB} = \langle -2-1, 1-0, 3-1 \rangle = \langle -3, 1, 2 \rangle$ and $\vec{AC} = \langle 3, 2, 4 \rangle$,

so a normal vector is given by their cross product:

$$\hat{\mathbf{n}} = \langle -3, 1, 2 \rangle \times \langle 3, 2, 4 \rangle = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = \langle 4-4, -(-12-6), -6-3 \rangle \\ = \langle 0, 18, -9 \rangle$$

We can use $\frac{1}{9} \hat{\mathbf{n}} = \langle 0, 2, -1 \rangle$ instead. The plane equation is

$$\langle 0, 2, -1 \rangle \cdot \langle x, y, z \rangle = \langle 0, 2, -1 \rangle \cdot \langle 1, 0, 1 \rangle$$

$$\Rightarrow \boxed{2y - z = -1}$$