Name: Key
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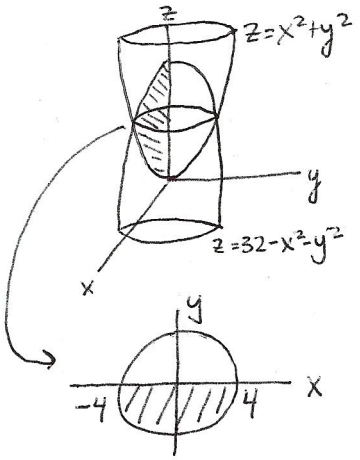
MAC 2313.8443
Cyr

Quiz 10
You must show all work to receive full credit!!

Problem 1. (4 pts) Use cylindrical coordinates to set up the triple integral that would be used to calculate the volume of the region between the paraboloids $z = x^2 + y^2$ and $z = 32 - x^2 - y^2$ with $y \leq 0$.



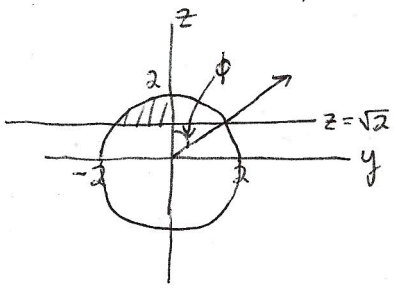
Intersection: $x^2+y^2 = 32-x^2-y^2 \Rightarrow 2x^2+2y^2 = 32$
$\Rightarrow x^2+y^2 = 16$

$x^2+y^2 = r^2$, $32-(x^2+y^2) = 32-r^2$

$$\int_{\pi}^{2\pi}\int_{0}^{4}\int_{r^2}^{32-r^2} r\,dz\,dr\,d\theta$$

$\pi \leq \theta \leq 2\pi$
$0 \leq r \leq 4$

Problem 2. (6 pts) Rewrite but do NOT evaluate the integral $\iiint_W z^{-1}dV$ using spherical coordinates, where $W = \{(x,y,z) \mid x^2+y^2+z^2 \leq 4, z \geq \sqrt{2}, y \leq 0, x \leq 0\}$.

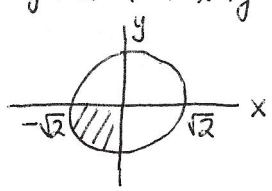


$z \geq \sqrt{2} \Rightarrow \rho\cos\phi \geq \sqrt{2} \Rightarrow \rho \geq \frac{\sqrt{2}}{\cos\phi}$.
$\rho^2 \leq 4 \Rightarrow \rho \leq 2$

When plane $z=\sqrt{2}$ intersects sphere $\rho = 2$ we have
$\rho\cos\phi = \sqrt{2} \Rightarrow 2\cos\phi = \sqrt{2} \Rightarrow \cos\phi = \frac{\sqrt{2}}{2} \Rightarrow \phi = \frac{\pi}{4}$

Integrand: $\frac{dV}{z} = \frac{\rho^2\sin\phi}{\rho\cos\phi}\,d\rho\,d\phi\,d\theta = \rho\tan\phi\,d\rho\,d\phi\,d\theta$

Intersection curve is
$x^2+y^2+2 \leq 4 \Rightarrow x^2+y^2 \leq 2$



$\Rightarrow \pi \leq \theta \leq \frac{3\pi}{2}$

$$\int_{\pi}^{3\pi/2}\int_{0}^{\pi/4}\int_{\frac{\sqrt{2}}{\cos\phi}}^{2} \rho\tan\phi\,d\rho\,d\phi\,d\theta$$