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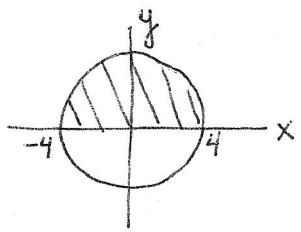
Quiz 10
You must show all work to receive full credit!!

Problem 1. (4 pts) On last week's quiz, you wrote (or should have written) the following integral using rectangular coordinates. Rewrite the integral by switching to cylindrical coordinates (do NOT evaluate).

$$\int_{-4}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{4} y\,dzdydx$$



$y = r\sin\theta \quad dzdydx = r\,dz\,dr\,d\theta$

$\Rightarrow$ integrand is $r^2\sin\theta\,dz\,dr\,d\theta$

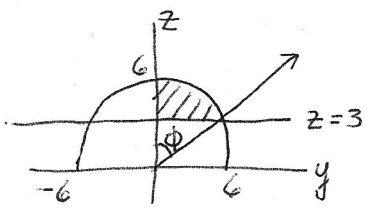
$$\int_0^{\pi}\int_0^{4}\int_r^{4} r^2\sin\theta\,dz\,dr\,d\theta$$

$0 \le r \le 4$

$0 \le \theta \le \pi$

$\sqrt{x^2+y^2} = \sqrt{r^2} = r$

Problem 2. (6 pts) Rewrite but do NOT evaluate the integral $\iiint_W x(x^2+y^2+z^2)^{-1/2}dV$ using spherical coordinates, where $W = \{(x,y,z)\,|\,x^2+y^2+z^2 \le 36, z \ge 3, y \ge 0, x \le 0\}$.



$z = 3 \Rightarrow \rho\cos\phi = 3$

Since $x^2+y^2+z^2 \le 36 \Rightarrow \rho^2 \le 36 \Rightarrow \rho \le 6$,

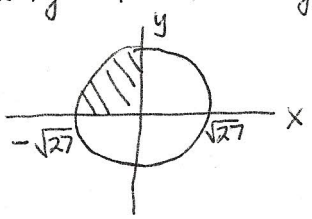
when $\rho = 6$ we have $6\cos\phi = 3 \Rightarrow \cos\phi = \frac{3}{6} = \frac{1}{2}$,

so $\cos\phi = \frac{\pi}{3}$. Thus, $0 \le \phi \le \frac{\pi}{3}$.

Finally, $z \ge 3 \Rightarrow \rho\cos\phi \ge 3 \Rightarrow \rho \ge \frac{3}{\cos\phi}$.

Integrand: $x(x^2+y^2+z^2)^{-1/2}dV = \rho\sin\phi\cos\theta\,(\rho^2)^{-1/2}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

The x,y-trace of intersection is $x^2+y^2+9 \le 36 \Rightarrow x^2+y^2 \le 27$



so $\frac{\pi}{2} \le \theta \le \pi$

$$\int_{\pi/2}^{\pi}\int_0^{\pi/3}\int_{\frac{3}{\cos\phi}}^{6} \rho^2\sin^2\phi\cos\theta\,d\rho\,d\phi\,d\theta$$