

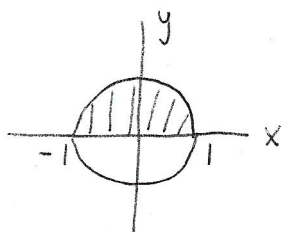
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Quiz 10

You must show all work to receive full credit!!

Problem 1. Evaluate $\iint_D y(x^2 + y^2)^3 dA$ using polar coordinates, where

$$D = \{(x, y) \mid x^2 + y^2 \leq 1, y \geq 0\}.$$



$$0 \leq r \leq 1$$

$$0 \leq \theta \leq \pi$$

$$\begin{aligned} y(x^2 + y^2)^3 &= (r \sin \theta)(r^2)^3 \\ &= (r \sin \theta)(r^6) \\ &= r^7 \sin \theta \end{aligned}$$

$$\begin{aligned} &\int_0^\pi \int_0^1 r^7 \sin \theta \, r \, dr \, d\theta \\ &= \int_0^\pi \int_0^1 r^8 \sin \theta \, dr \, d\theta \\ &= \int_0^\pi \left. \frac{r^9}{9} \sin \theta \right|_0^1 d\theta = \int_0^\pi \frac{1}{9} \sin \theta \, d\theta \\ &= -\frac{1}{9} \cos \theta \Big|_0^\pi = -\frac{1}{9}(-1 - 1) = -\frac{1}{9}(-2) = \boxed{\frac{2}{9}} \end{aligned}$$

Problem 2. Evaluate $\iiint_W \rho^{-1} dV$ using spherical coordinates, where

$$W = \{(x, y, z) \mid 3 \leq x^2 + y^2 + z^2 \leq 9\}.$$

$$\begin{aligned} 3 \leq \rho^2 \leq 9 &\Rightarrow \sqrt{3} \leq \rho \leq 3 \\ 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \pi &\Rightarrow \int_0^\pi \int_0^{2\pi} \int_{\sqrt{3}}^3 \rho^{-1} \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \\ &= \int_0^\pi \int_0^{2\pi} \int_{\sqrt{3}}^3 \rho \sin \phi \, d\rho \, d\theta \, d\phi \end{aligned}$$

$$\begin{aligned} &= \int_0^\pi \int_0^{2\pi} \left. \frac{\rho^2}{2} \right|_{\sqrt{3}}^3 \sin \phi \, d\theta \, d\phi = \int_0^\pi \int_0^{2\pi} \frac{9-3}{2} \sin \phi \, d\theta \, d\phi = 3 \int_0^\pi \sin \phi \cdot \theta \Big|_0^{2\pi} d\phi \\ &= 6\pi \int_0^\pi \sin \phi \, d\phi = 6\pi [-\cos \phi]_0^\pi = 6\pi(1+1) = \boxed{12\pi} \end{aligned}$$