

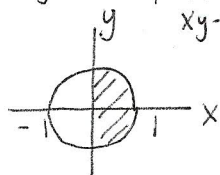
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 March 26, 2015  
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### Quiz 10

You must show all work to receive full credit!!

**Problem 1.** Evaluate  $\iiint_W xz dV$  using cylindrical coordinates, where

Projection of  $W$  in  
 $xy$ -plane:



$$0 \leq r \leq 1$$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$W = \{(x, y, z) \mid x^2 + y^2 \leq 1, x \geq 0, 0 \leq z \leq 2\}.$$

$$\begin{aligned} \int_{-\pi/2}^{\pi/2} \int_0^1 \int_0^2 (r \cos \theta) z r dz dr d\theta &= \int_{-\pi/2}^{\pi/2} \int_0^1 \int_0^2 z r^2 \cos \theta dz dr d\theta \\ &= \left( \int_{-\pi/2}^{\pi/2} \cos \theta d\theta \right) \left( \int_0^1 r^2 dr \right) \left( \int_0^2 z dz \right) \\ &= \left( \sin \theta \Big|_{-\pi/2}^{\pi/2} \right) \left( \frac{r^3}{3} \Big|_0^1 \right) \left( \frac{z^2}{2} \Big|_0^2 \right) \\ &= (1+1) \left( \frac{1}{3} \right) (2) = (2) \left( \frac{1}{3} \right) (2) = \left( \frac{4}{3} \right) \end{aligned}$$

**Problem 2.** Evaluate  $\iiint_W \rho^{-3} dV$  using spherical coordinates, where

$$W = \{(x, y, z) \mid 2 \leq x^2 + y^2 + z^2 \leq 4\}.$$

$$2 \leq \rho^2 \leq 4 \Rightarrow \sqrt{2} \leq \rho \leq 2$$

$$0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi$$

$$\begin{aligned} &\Rightarrow \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^2 \rho^{-3} \rho^2 \sin \phi d\rho d\phi d\theta \\ &= \int_0^{2\pi} \int_0^\pi \int_{\sqrt{2}}^2 \frac{1}{\rho} \sin \phi d\rho d\phi d\theta \end{aligned}$$

$$= \left( \int_0^{2\pi} d\theta \right) \left( \int_0^\pi \sin \phi d\phi \right) \left( \int_{\sqrt{2}}^2 \frac{1}{\rho} d\rho \right)$$

$$= \left( \theta \Big|_0^{2\pi} \right) \left( -\cos \phi \Big|_0^\pi \right) \left( \ln \rho \Big|_{\sqrt{2}}^2 \right)$$

$$= (2\pi) (1+1) (\ln 2 - \ln \sqrt{2}) = \boxed{4\pi (\ln 2 - \ln \sqrt{2})}$$

$$\text{OR } \boxed{2\pi \ln(2)}$$