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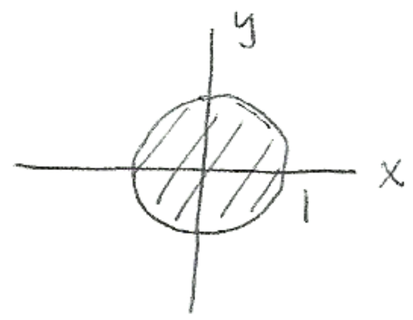
MAC 2313.6717
Cyr

Quiz 10
You must show all work to receive full credit!!

Problem 1. (5 points) Use polar coordinates to find the volume of the solid below the plane $2x + y + z = 4$ and above the disk $x^2 + y^2 \leq 1$.



$z = 4 - 2x - y \Rightarrow z = 4 - 2r\cos\theta - r\sin\theta$

$$V = \int_0^{2\pi}\int_0^1 (4 - 2r\cos\theta - r\sin\theta)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 (4r - 2r^2\cos\theta - r^2\sin\theta)\, dr\, d\theta$$

$$= \int_0^{2\pi} 2r^2 - \frac{2}{3}r^3\cos\theta - \frac{1}{3}r^3\sin\theta \Big|_0^1 d\theta$$

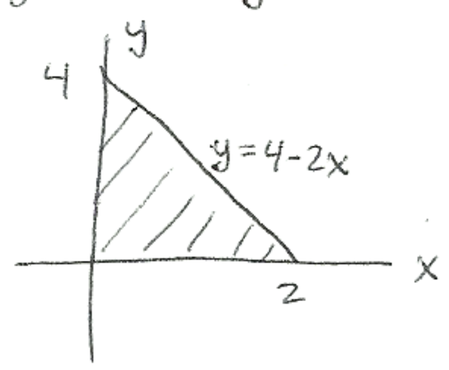
$$= \int_0^{2\pi} \left(2 - \frac{2}{3}\cos\theta - \frac{1}{3}\sin\theta\right) d\theta$$

$$= 2\theta - \frac{2}{3}\sin\theta + \frac{1}{3}\cos\theta \Big|_0^{2\pi} = \left(4\pi + \frac{1}{3}\right) - \left(\frac{1}{3}\right) = \boxed{4\pi}$$

Problem 2. (5 points) Use a triple integral to find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\, dy\, dx$$

Intersection in xy-plane:
$2x + y = 4 \Rightarrow y = 4 - 2x$



$$= \int_0^2 \int_0^{4-2x} (4 - 2x - y)\, dy\, dx$$

$$= \int_0^2 (4-2x)\, y - \frac{1}{2}y^2 \Big|_0^{4-2x} dx = \int_0^2 \left[(4-2x)^2 - \frac{1}{2}(4-2x)^2\right] dx$$

$$= \frac{1}{2}\int_0^2 (4-2x)^2 dx = \frac{1}{2}\int_0^2 (16 - 16x + 4x^2)\, dx$$

$$= \int_0^2 (8 - 8x + 2x^2)\, dx = 8x - 4x^2 + \frac{2}{3}x^3 \Big|_0^2$$

$$= 16 - 16 + \frac{2}{3}(8) = \boxed{\frac{16}{3}}$$