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Quiz 10

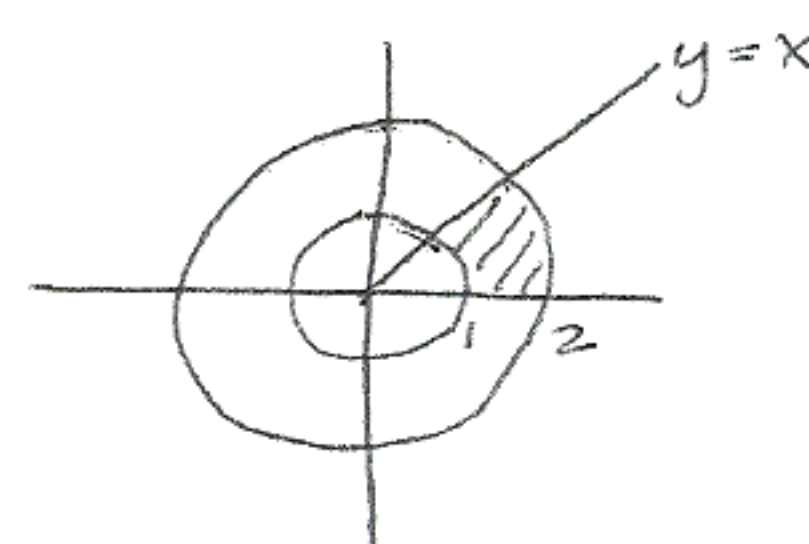
You must show all work to receive full credit!!

Problem 1. (5 points) Use polar coordinates to evaluate $\iint_D \arctan\left(\frac{y}{x}\right) dA$, where $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

$$y = x \Leftrightarrow r \sin \theta = r \cos \theta \Rightarrow \tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\tan \theta = \frac{y}{x} \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$$

$$\begin{aligned} \int_0^{\pi/4} \int_1^2 \theta r dr d\theta &= \left(\int_0^{\pi/4} \theta d\theta \right) \left(\int_1^2 r dr \right) \\ &= \left(\frac{1}{2} \theta^2 \Big|_0^{\pi/4} \right) \left(\frac{1}{2} r^2 \Big|_1^2 \right) \\ &= \frac{\pi^2}{32} \cdot \left(2 - \frac{1}{2} \right) = \frac{\pi^2}{32} \cdot \frac{3}{2} \\ &= \boxed{\frac{3\pi^2}{64}} \end{aligned}$$



Problem 2. (5 points) Evaluate $\iiint_E xy dV$, where E lies under the plane $z = 8 + 8x + 3y$ and above the region in the xy -plane bounded by the curves $y = x$, $y = 0$, and $x = 1$.

$$\begin{aligned} &\int_0^1 \int_0^x \int_0^{8+8x+3y} xy dz dy dx \\ &= \int_0^1 \int_0^x (8xy + 8x^2y + 3xy^2) dy dx \\ &= \int_0^1 (4xy^2 + 4x^2y^2 + xy^3) \Big|_0^x dx \\ &= \int_0^1 (4x^3 + 4x^4 + x^4) dx = \int_0^1 (4x^3 + 5x^4) dx \\ &= x^4 + x^5 \Big|_0^1 = 1 + 1 = \textcircled{2} \end{aligned}$$

