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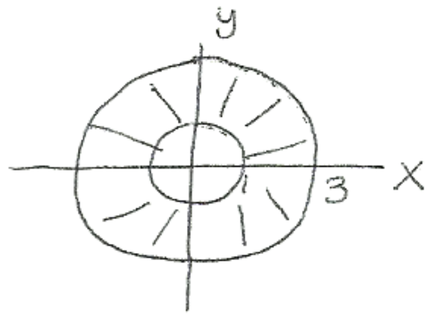
Quiz 10
You must show all work to receive full credit!!

Problem 1. (5 points) Use polar coordinates to evaluate $\iint_D \frac{y^2}{x^2+y^2} dA$, where $D = \{(x,y) \,|\, 1 \le x^2+y^2 \le 9\}$.



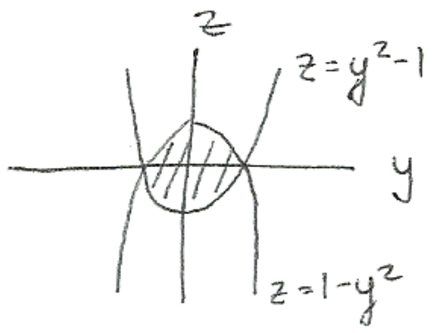
$$\int_0^{2\pi}\int_1^3 \frac{r^2\sin^2\theta}{r^2} r\,dr\,d\theta = \int_0^{2\pi}\int_1^3 r\sin^2\theta\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}r^2\Big|_1^3 \sin^2\theta\,d\theta = \int_0^{2\pi} 4\sin^2\theta\,d\theta$$

$$= \int_0^{2\pi} 2(1-\cos(2\theta))\,d\theta = 2\theta - \sin(2\theta)\Big|_0^{2\pi}$$

$$= \boxed{4\pi}$$

Problem 2. (5 points) Evaluate $\iiint_E (x-y)dV$, where E is enclosed by the surfaces $z = y^2-1, z = 1-y^2, x = 0$, and $x = 1$.



Intersect at $y^2-1 = 1-y^2 \Rightarrow 2y^2 = 2 \Rightarrow y^2 = 1$
$\Rightarrow y = \pm 1$. So

$$\int_0^1\int_{-1}^1\int_{y^2-1}^{1-y^2} (x-y)\,dz\,dy\,dx = \int_0^1\int_{-1}^1 (x-y)[1-y^2-(y^2-1)]\,dy\,dx$$

$$= \int_0^1\int_{-1}^1 (2x-2y-2xy^2+2y^3)\,dy\,dx = \int_0^1 2xy - y^2 - \frac{2}{3}xy^3 + \frac{1}{2}y^4\Big|_{-1}^1 dx$$

$$= \int_0^1 \left[\left(2x - x - \frac{2}{3}x + \frac{1}{2}\right) - \left(-2x - 1 + \frac{2}{3}x + \frac{1}{2}\right)\right] dx = \int_0^1 \left(4x - \frac{4}{3}x\right) dx$$

$$= \int_0^1 \frac{8}{3}x\,dx = \frac{4}{3}x^2\Big|_0^1 = \boxed{\frac{4}{3}}$$