Name: Key
July 30, 2015
MAC 2313.8326
Cyr

Quiz 11
You must show all work to receive full credit!!

**Problem 1.** (3 pts) Let $\mathbf{F} = \langle 2xy+5, x^2-4z, -4y\rangle$. Given that $\mathbf{F}$ is conservative, evaluate $\int_C \mathbf{F}d\mathbf{r}$, where $\mathcal{C}$ is any curve from $(1,1,0)$ to $(4,-2,1)$.

$\int (2xy+5)dx = x^2y + 5x + f(y,z)$

$\int (x^2-4z)dy = x^2y - 4yz + g(x,z)$

$\int (-4y)dz = -4yz + h(x,y)$

$\Rightarrow \phi(x,y,z) = x^2y + 5x - 4yz$

Then $\int_C \vec{F}d\vec{r} = \phi(4,-2,1) - \phi(1,1,0)$

$= [16(-2) + 5(4) - 4(-2)(1)] - [1\cdot 1 + 5\cdot 1 - 4\cdot 1\cdot 0]$

$= (-32+20+8) - (1+5)$

$= -4-6 = \boxed{-10}$

**Problem 2.** (3 pts) Calculate $\iint_S z ds$ where $\mathcal{S}$ is the part of the surface $z = \frac{2}{3}x^3$ with $0 \le x \le 2$, $0 \le y \le 9$.

$z = K(x,y) = \frac{2}{3}x^3 \Rightarrow K_x = 2x^2, K_y = 0$. Then $\|\vec{n}\| = \sqrt{1+K_x^2+K_y^2} = \sqrt{1+(2x^2)^2+0^2} = \sqrt{1+4x^4}$.

So $\iint_S z\,dS = \int_0^2\int_0^9 \frac{2}{3}x^3\sqrt{1+4x^4}\,dy\,dx = 6\int_0^2 x^3\sqrt{1+4x^4}\,dx$

$u = 1+4x^4$
$du = 16x^3dx$

$= \frac{6}{16}\int_1^{65} u^{1/2}du = \frac{1}{4}u^{3/2}\Big|_1^{65}$

$= \boxed{\frac{1}{4}(65^{3/2}-1)}$

**Problem 3.** (4 pts) Set up the integral $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \langle y, z, 0\rangle$ and $\mathcal{S}$ has parametrization $\mathbf{r}(u,v) = \langle u^3 - v, u+v, v^2\rangle$ for $0 \le u \le 2$, $0 \le v \le 3$ with downward-pointing normal. (Do NOT evaluate.)

$\vec{r}_u = \langle 3u^2, 1, 0\rangle$, $\vec{r}_v = \langle -1, 1, 2v\rangle$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3u^2 & 1 & 0 \\ -1 & 1 & 2v \end{vmatrix}$$

$= \langle 2v, -6u^2v, 3u^2+1\rangle$

$z > 0$ for all $u$, so wrong orientation

Take $\vec{n} = \langle -2v, 6u^2v, -3u^2-1\rangle$.

$\vec{F} = \langle y, z, 0\rangle = \langle u+v, v^2, 0\rangle$, so

$\vec{F}\cdot\vec{n} = \langle u+v, v^2, 0\rangle \cdot \langle -2v, 6u^2v, -3u^2-1\rangle$

$= -2uv - 2v^2 + 6u^2v^3$

Then $\boxed{\iint_S \vec{F}d\vec{S} = \int_0^3\int_0^2 (-2uv - 2v^2 + 6u^2v^3)\,du\,dv}$