Name: Key
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MAC 2313.6717
Cyr

Quiz 11
You must show all work to receive full credit!!

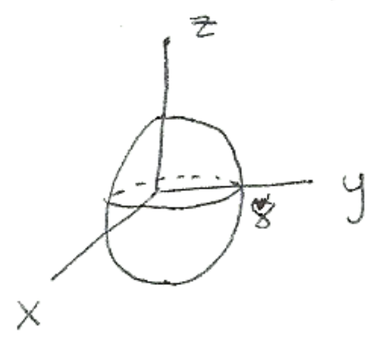
Problem 1. (5 points) Rewrite the integral $\iiint_E y dV$ in spherical coordinates, where E is the portion of the ball $x^2+y^2+z^2 \leq 64$ where $x \leq 0, y \leq 0, z \leq 0$. (Do NOT evaluate.)

$x, y \leq 0 \Rightarrow \theta \in QIII$

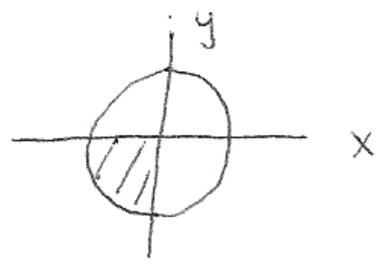
$z \leq 0 \Rightarrow \frac{\pi}{2} \leq \phi \leq \pi$

$x^2+y^2+z^2 \leq 64 \Rightarrow \rho^2 \leq 64 \Rightarrow \rho \leq 8.$

$y = \rho \sin\theta \sin\phi$



$$\int_{\pi}^{3\pi/2} \int_{\pi/2}^{\pi} \int_{0}^{8} (\rho \sin\theta \sin\phi)(\rho^2 \sin\phi)\, d\rho\, d\phi\, d\theta$$



Problem 2. (5 points) Use the transformation $T(u,v) = (6u+3v, -u-v)$ to evaluate the integral $\iint_D (x+y) dA$, given that $D = T(S)$ and $S = \{(u,v) \mid 0 \leq v \leq u, 0 \leq u \leq 2\}$.

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 6 & 3 \\ -1 & -1 \end{vmatrix} = -6 - (-3) = -3$$

$x + y = 6u + 3v - u - v = 5u + 2v$

$$\iint_D (x+y) dA = \int_0^2 \int_0^u (5u+2v)\,|-3|\, dv\, du = 3\int_0^2 \int_0^u (5u+2v)\, dv\, du$$

$$= 3\int_0^2 5uv + v^2 \Big|_0^u du = 3\int_0^2 (5u^2+u^2)\, du = 3\int_0^2 6u^2\, du$$

$$= 3\left(2u^3\Big|_0^2\right) = 3(16) = \boxed{48}$$