Name: Key
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MAC 2313.9722
Cyr

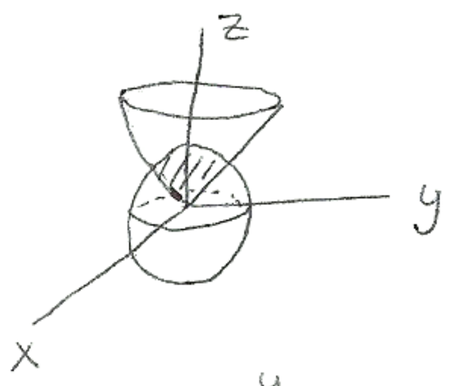
Quiz 11
You must show all work to receive full credit!!

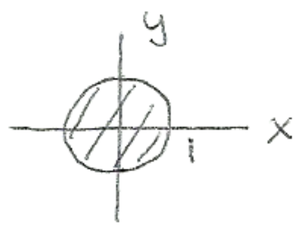
Problem 1. (5 points) Use cylindrical coordinates to write a triple integral which gives the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2+y^2+z^2 = 2$. (Do NOT evaluate.)

$z = \sqrt{x^2+y^2} = \sqrt{r^2} = r$

$z^2 = 2 - x^2 - y^2 = 2 - r^2 \Rightarrow z = \sqrt{2-r^2}$

Intersect at $x^2+y^2 + (x^2+y^2) = 2 \Rightarrow x^2+y^2 = 1$





$$V = \int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$$

Problem 2. (5 points) Use the transformation $T(u, v) = (u + v, v - u)$ to evaluate $\iint_D \frac{x-y}{x+y} dA$, given that $D = T(S)$ and $S = \{(u, v) \mid -2 \le u \le 0, 1 \le v \le 2\}$.

$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 - (-1) = 2$$

$x - y = u + v - (v - u) = 2u$

$x + y = u + v + v - u = 2v$

$\Rightarrow \frac{x-y}{x+y} = \frac{2u}{2v} = uv^{-1}$

$$\iint_D \frac{x-y}{x+y} dA = \int_1^2 \int_{-2}^0 2uv^{-1} \, du \, dv = \int_1^2 u^2 v^{-1} \Big|_{-2}^0 \, dv$$

$$= \int_1^2 -\frac{4}{v} dv = -4 \ln v \Big|_1^2 = \boxed{-4 \ln(2)}$$