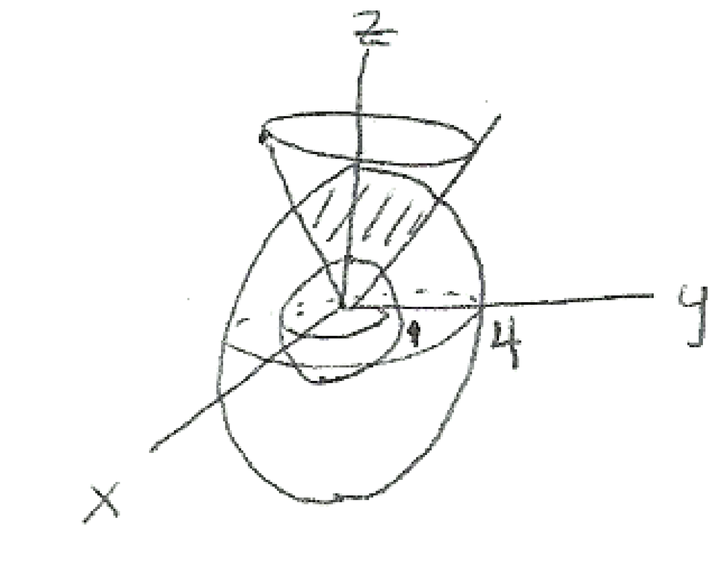
Name: Key
November 10, 2016
MAC 2313.9728
Cyr

Quiz 11
You must show all work to receive full credit!!

Problem 1. (5 points) Rewrite the integral $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$ in spherical coordinates, where E lies above the cone $z=\sqrt{x^2+y^2}$ and between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=16$. (Do NOT evaluate.)



$1 \le x^2+y^2+z^2 \le 16 \Rightarrow 1 \le \rho^2 \le 16 \Rightarrow 1 \le \rho \le 4.$

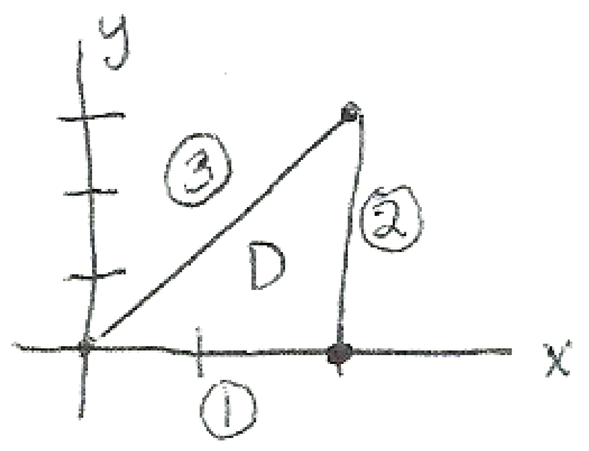
$z=\sqrt{x^2+y^2} \Rightarrow \rho\cos\phi = \sqrt{\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)} = \rho\sin\phi$

$\Rightarrow \tan\phi = 1 \Rightarrow \phi = \frac{\pi}{4}$

$\sqrt{x^2+y^2+z^2} = \sqrt{\rho^2} = \rho$

$$\boxed{\int_0^{2\pi}\int_0^{\pi/4}\int_1^4 \rho\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta}$$

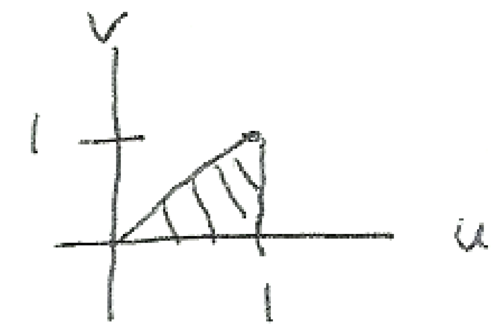
Problem 2. (5 points) Rewrite the integral $\iint_D x^2\,dA$ by using the transformation $T(u,v)=(2u,3v)$, where D is the triangular region with vertices $(0,0)$, $(2,0)$, and $(2,3)$.



① $y=0 \Rightarrow 3v=0 \Rightarrow v=0$

② $x=2 \Rightarrow 2u=2 \Rightarrow u=1$

③ $y=\frac{3}{2}x \Rightarrow 3v=3u \Rightarrow v=u$



$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 \qquad x^2 = (2u)^2 = 4u^2$$

$$\iint_D x^2\,dA = \boxed{\int_0^1\int_0^u 4u^2\cdot 6\,dv\,du}$$