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Quiz 12
You must show all work to receive full credit!!

Problem 1. (4 pts) Compute $\int_C f ds$ where $f(x,y,z) = 2x^2+8z$ and C has the parametrization $\mathbf{r}(t) = \langle e^t, t^2, t\rangle$ for $0 \le t \le 1$.

$\vec{r}\,'(t) = \langle e^t, 2t, 1\rangle \Rightarrow \|\vec{r}\,'(t)\| = \sqrt{e^{2t}+4t^2+1}$

$f(t) = 2(e^t)^2 + 8t = 2e^{2t} + 8t$

$\int_C f ds = \int_0^1 f(t)\|\vec{r}\,'(t)\| dt = \int_0^1 (2e^{2t}+8t)\sqrt{e^{2t}+4t^2+1}\, dt$

$u = e^{2t}+4t^2+1$
$du = (2e^{2t}+8t)dt$

$= \int_2^{e^2+5} u^{1/2} du = \frac{2}{3}u^{3/2}\Big|_2^{e^2+5}$

$= \boxed{\frac{2}{3}\left[(e^2+5)^{3/2} - 2^{3/2}\right]}$

Problem 2. (6 pts) Evaluate $\int_C \mathbf{F} d\mathbf{r}$, where $\mathbf{F} = \langle z\sec^2 x, z, y+\tan x\rangle$ and C is a path from $(0,1,2)$ to $(\frac{\pi}{4}, 2, 4)$. (Hint: do not parameterize C.)

Check if $\vec{F}$ is conservative: $\frac{\partial}{\partial y}(z\sec^2 x) = 0 = \frac{\partial}{\partial x}(z)$, $\frac{\partial}{\partial z}(z) = 1 = \frac{\partial}{\partial y}(y+\tan x)$,
$\frac{\partial}{\partial x}(y+\tan x) = \sec^2 x = \frac{\partial}{\partial z}(z\sec^2 x) \Rightarrow \vec{F}$ is conservative.

Find a potential function: $\int z\sec^2 x\, dx = z\tan x + f(y,z)$
$\int z\, dy = yz + g(x,z)$
$\int (y+\tan x) dz = yz + z\tan x + h(x,y)$

$\Rightarrow \phi(x,y,z) = yz + z\tan x$
satisfies $\nabla\phi = \vec{F}$

Therefore, by Fundamental Thrm of Conservative Vector Fields,

$\int_C \vec{F} d\vec{r} = \phi(\frac{\pi}{4}, 2, 4) - \phi(0,1,2) = [8 + 4\cdot 1] - [2+0]$
$= 12 - 2 = \boxed{10}$