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MAC 2313.3118
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Quiz 12
You must show all work to receive full credit!!

Problem 1. (5 pts) Compute $\int_C f ds$ where $f(x, y, z) = 3x - 2y + z$ and C has parametrization $\mathbf{c}(t) = (2 + t, 2 - t, 2t)$ for $-2 \leq t \leq 1$.

$$\int_C f ds = \int_a^b f(\vec{c}(t)) \, \|\vec{c}\,'(t)\| dt$$

We find $f(\vec{c}(t)) = 3(2+t) - 2(2-t) + 2t = 6 + 3t - 4 + 2t + 2t = 7t + 2$;

$\vec{c}\,'(t) = (1, -1, 2) \Rightarrow \|\vec{c}\,'(t)\| = \sqrt{1+1+4} = \sqrt{6}$.

So $\int_C f ds = \sqrt{6} \int_{-2}^{1} (7t+2) dt = \sqrt{6} \left(\frac{7}{2}t^2 + 2t \right) \Big|_{-2}^{1}$

$= \sqrt{6} \left[\left(\frac{7}{2} + 2 \right) - (14 - 4) \right] = \sqrt{6} \left(\frac{11}{2} - \frac{20}{2} \right) = \boxed{\frac{-9\sqrt{6}}{2}}$

Problem 2. (5 pts) Compute $\int_C \mathbf{F} d\mathbf{s}$ where $\mathbf{F} = \left\langle \frac{1}{y^3 + 1}, \frac{1}{z + 1}, 1 \right\rangle$ and C is the oriented curve with parametrization $\mathbf{c}(t) = (t^3, 2, t^2)$ for $0 \leq t \leq 1$.

$$\int_C \vec{F} d\vec{s} = \int_a^b \vec{F}(\vec{c}(t)) \cdot \vec{c}\,'(t) \, dt$$

We find $\vec{F}(\vec{c}(t)) = \left\langle \frac{1}{9}, \frac{1}{t^2+1}, 1 \right\rangle$ and $\vec{c}\,'(t) = \langle 3t^2, 0, 2t \rangle$, so

$\vec{F}(\vec{c}(t)) \cdot \vec{c}\,'(t) = \frac{1}{3}t^2 + 2t$.

$\int_C \vec{F} d\vec{s} = \int_0^1 \left(\frac{1}{3}t^2 + 2t \right) dt = \frac{1}{9}t^3 + t^2 \Big|_0^1 = \frac{1}{9} + 1 = \boxed{\frac{10}{9}}$