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MAC 2313.3122
Cyr

Quiz 12

You must show all work to receive full credit!!

Problem 1. (5 pts) Compute $\int_C f ds$ where $f(x, y) = \sqrt{1 + 9xy}$ and C has parametrization $\mathbf{c}(t) = (t, t^3)$ for $0 \le t \le 1$.

$$\int_C f ds = \int_a^b f(\vec{c}(t)) \|\vec{c}\,'(t)\| dt$$

We find $f(\vec{c}(t)) = \sqrt{1+9t\cdot t^3} = \sqrt{1+9t^4}$ and $\vec{c}\,'(t) = (1, 3t^2) \Rightarrow$
$\|\vec{c}\,'(t)\| = \sqrt{1+(3t^2)^2} = \sqrt{1+9t^4}$. Therefore,

$$\int_C f ds = \int_0^1 \sqrt{1+9t^4}\sqrt{1+9t^4}\, dt = \int_0^1 (1+9t^4)\, dt = t + \frac{9}{5}t^5 \Big|_0^1$$

$$= 1 + \frac{9}{5} = \frac{14}{5}$$

Problem 2. (5 pts) Compute $\int_C \mathbf{F} ds$ where $\mathbf{F} = \langle 3zy^{-1}, 4x, -y \rangle$ and C is the oriented curve with parametrization $\mathbf{c}(t) = (e^t, e^t, t)$ for $0 \le t \le 1$.

$$\int_C \vec{F} d\vec{s} = \int_a^b \vec{F}(\vec{c}(t)) \cdot \vec{c}\,'(t)\, dt$$

We find $\vec{F}(\vec{c}(t)) = \langle \frac{3t}{e^t}, 4e^t, -e^t \rangle$ and $\vec{c}\,'(t) = \langle e^t, e^t, 1 \rangle$, so
$\vec{F}(\vec{c}(t)) \cdot \vec{c}\,'(t) = 3t + 4e^{2t} - e^t$. Therefore,

$$\int_C \vec{F}\cdot d\vec{s} = \int_0^1 (3t + 4e^{2t} - e^t)\, dt = \frac{3}{2}t^2 + 2e^{2t} - e^t \Big|_0^1$$

$$= \left(\frac{3}{2} + 2e^2 - e\right) - (2-1) = \boxed{\frac{1}{2} + 2e^2 - e}$$