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MAC 2313.8326
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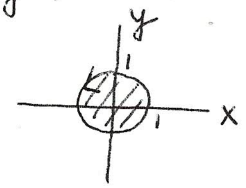
Quiz 12

You must show all work to receive full credit!!

Problem 1. (5 pts) Use Green's Theorem to evaluate $\oint_C xy^2\,dy$, where C is the unit circle centered at the origin, oriented counterclockwise.

$\vec{F} = \langle f, g\rangle = \langle 0, xy^2\rangle$



$$\oint_C \vec{F}\cdot d\vec{r} = \iint_D (g_x - f_y)\,dA = \iint_D (y^2 - 0)\,dA$$

$$= \int_0^{2\pi}\int_0^1 (r\sin\theta)^2\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 r^3\sin^2\theta\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^4}{4}\Big|_0^1 \sin^2\theta\,d\theta = \frac{1}{4}\int_0^{2\pi}\sin^2\theta\,d\theta = \frac{1}{8}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta$$

$$= \frac{1}{8}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{2\pi} = \frac{1}{8}\left[(2\pi - 0) - (0 - 0)\right] = \boxed{\frac{\pi}{4}}$$

Problem 2. (3 pts) Let C be a simple, closed curve enclosing a region D such that the conditions of Green's Theorem are satisfied. Justify or disprove the claim that the area of D can be calculated by evaluating $\oint_C (y\tan^2 x)dx + (\tan x + e^{y^3})dy$.

$\vec{F} = \langle f, g\rangle = \langle y\tan^2 x, \tan x + e^{y^3}\rangle \Rightarrow g_x - f_y = \sec^2 x - \tan^2 x.$

Since $\sec^2 x = \tan^2 x + 1 \Rightarrow \sec^2 x - \tan^2 x = 1$, $g_x - f_y = 1$. Thus, by Green's Theorem, $\oint_C \vec{F}\cdot d\vec{r} = \iint_D 1\,dA = \text{Area}(D)$.

Claim is true.

Problem 3. (2 pts) Is the following statement true or false? Every conservative vector field is irrotational.

True

[If $\vec{F} = \langle f, g\rangle$ is conservative, then $f_y = g_x$, which is the definition of an irrotational vector field.]