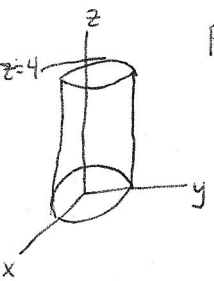
Name: Key
April 14, 2016
MAC 2313.8443
Cyr

Quiz 13
You must show all work to receive full credit!!

Problem 1. (5 pts) Set up (but do NOT evaluate) $\iint_S xe^z\,dS$, where $\mathcal{S}$ is the portion of the cylinder $x^2+y^2=4$ with $0\le z\le 4$.



Parametrize S by $\vec{r}(\theta,z) = \langle 2\cos\theta, 2\sin\theta, z\rangle$ where $0\le\theta\le 2\pi$, $0\le z\le 4$.

Then $\vec{r}_\theta = \langle -2\sin\theta, 2\cos\theta, 0\rangle$ and $\vec{r}_z = \langle 0,0,1\rangle$, so

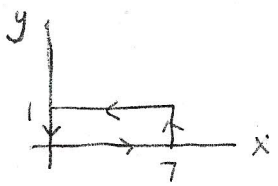
$$\vec{n} = \vec{r}_\theta \times \vec{r}_z = \begin{vmatrix} i & j & k \\ -2\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 2\cos\theta, 2\sin\theta, 0\rangle$$

and $\|\vec{n}\| = \sqrt{4\cos^2\theta + 4\sin^2\theta} = \sqrt{4} = 2$.

$f(\theta,z) = (2\cos\theta)e^z$, so

$$\iint_S xe^z\,dS = \int_0^{2\pi}\int_0^4 2\cos\theta e^z \cdot 2\,dz\,d\theta = \boxed{\int_0^{2\pi}\int_0^4 4\cos\theta e^z\,dz\,d\theta}$$

Problem 2. (5 pts) Evaluate $\oint_C \mathbf{F}d\mathbf{r}$, where $\mathbf{F} = \langle y^2, x^2\rangle$ and C is the boundary of the rectangular region $0\le x\le 7, 0\le y\le 1$, oriented counterclockwise. (Hint: use Green's Theorem.)



$\vec{F} = \langle y^2, x^2\rangle \Rightarrow g_x - f_y = 2x - 2y$

By Green's Thm, $\oint_C \vec{F}\,d\vec{r} = \iint_D (g_x - f_y)\,dA$

$$= \int_0^7\int_0^1 (2x-2y)\,dy\,dx = \int_0^7 (2xy - y^2)\Big|_0^1\,dx$$

$$= \int_0^7 (2x-1)\,dx = x^2 - x\Big|_0^7 = 49 - 7 = \boxed{42}$$