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MAC 2313.3118
Cyr

Quiz 13

You must show all work to receive full credit!!

Problem 1. (3 pts) Let $V(x,y,z) = xy\sin(yz)$ and $\mathbf{F} = \nabla V$. Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{c}$ is any path from $(0,0,0)$ to $(2, \frac{1}{2}, \pi)$.

Since $\vec{F}$ has potential function V, it is conservative, so

$$\int_{\vec{c}} \vec{F} \cdot d\vec{s} = V(2, \tfrac{1}{2}, \pi) - V(0,0,0)$$
$$= (2)(\tfrac{1}{2})\sin\tfrac{\pi}{2} - 0$$
$$= 1$$

Problem 2. (7 pts) Calculate $\iint_S z\,dS$ where the surface S is the portion of the cylinder $y = 9 - z^2$ where $0 \le x \le 3, 0 \le z \le 3$. (Hint: write a parametrization for S which depends on the variables x and z.)

A parametrization for S is $G(x,z) = (x, 9-z^2, z)$, for $0 \le x \le 3$, $0 \le z \le 3$.

Then $\vec{T}_x = \langle 1,0,0 \rangle$, $\vec{T}_z = \langle 0, -2z, 1 \rangle$, and $\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & -2z & 1 \end{vmatrix} = \langle 0, -1, -2z \rangle$.

Since we are finding a scalar surface integral, we calculate $\|\vec{n}\| = \sqrt{1+4z^2}$.

Then $\iint_S z\,dS = \int_0^3 \int_0^3 z\sqrt{1+4z^2}\,dx\,dz = 3\int_0^3 z\sqrt{1+4z^2}\,dz$ $u = 1+4z^2$

$= \frac{3}{8}\int_1^{37} u^{1/2}\,du = \frac{1}{4}u^{3/2}\Big|_1^{37}$ $du = 8z\,dz$

$= \boxed{\dfrac{37\sqrt{37}-1}{4}}$