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MAC 2313.3122
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Quiz 13

You must show all work to receive full credit!!

Problem 1. (3 pts) Let $V(x, y, z) = xye^z$ and $\mathbf{F} = \nabla V$. Evaluate $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{c}(t) = (t^2, t^3, t-1)$ for $1 \leq t \leq 2$.

Since $\vec{F}$ has a potential function V, it is conservative, so

$\int_{\vec{c}} \vec{F} \cdot d\vec{s} = V(\vec{c}(2)) - V(\vec{c}(1)) = V(4, 8, 1) - V(1, 1, 0)$

$= \boxed{32e - 1}$

Problem 2. (7 pts) Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = \langle \sin y, \sin z, yz \rangle$ and the surface S is the rectangle $0 \leq y \leq 2, 0 \leq z \leq 3$ in the (y, z)-plane, with normal vector pointing in the **negative** x-direction.

A parametrization for S is $G(y, z) = (0, y, z)$ with $0 \leq y \leq 2$, $0 \leq z \leq 3$ (since every pt in (y, z)-plane has x-coordinate 0). Then

$\vec{T_y} = \frac{\partial G}{\partial y} = \langle 0, 1, 0 \rangle = \hat{j}$ and $\vec{T_z} = \langle 0, 0, 1 \rangle = \hat{k}$ implies

$\vec{n} = \vec{T_y} \times \vec{T_z} = \hat{j} \times \hat{k} = \hat{i} = \langle 1, 0, 0 \rangle$. But orientation of S is in <u>negative</u> x-direction,

so take $\vec{n} = \langle -1, 0, 0 \rangle$. Then $\vec{F} \cdot \vec{n} = \langle \sin y, \sin z, yz \rangle \cdot \langle -1, 0, 0 \rangle = -\sin y$.

Then $\iint_S \vec{F} \cdot d\vec{S} = \int_0^2 \int_0^3 -\sin y \, dz\, dy = 3 \int_0^2 -\sin y \, dy = 3 \cos y \Big|_0^2$

$= \boxed{3\cos 2 - 3}$